Discovery Project

PREDATOR/PREY MODELS

Sine and cosine functions are used primarily in physics and engineering to model oscillatory behavior, such as the motion of a pendulum or the current in an AC electrical circuit. (See *Focus on Modeling*, pages 554–565.) But these functions also arise in the other sciences. In this project, we consider an application to biology—we use sine functions to model the population of a predator and its prey.

An isolated island is inhabited by two species of mammals: foxes and rabbits. The foxes are *predators* who feed on the rabbits, their *prey*. The fox and rabbit populations change cyclically, as graphed in Figure 1. In part A of the graph, rabbits are abundant, so the foxes have plenty to eat and their population

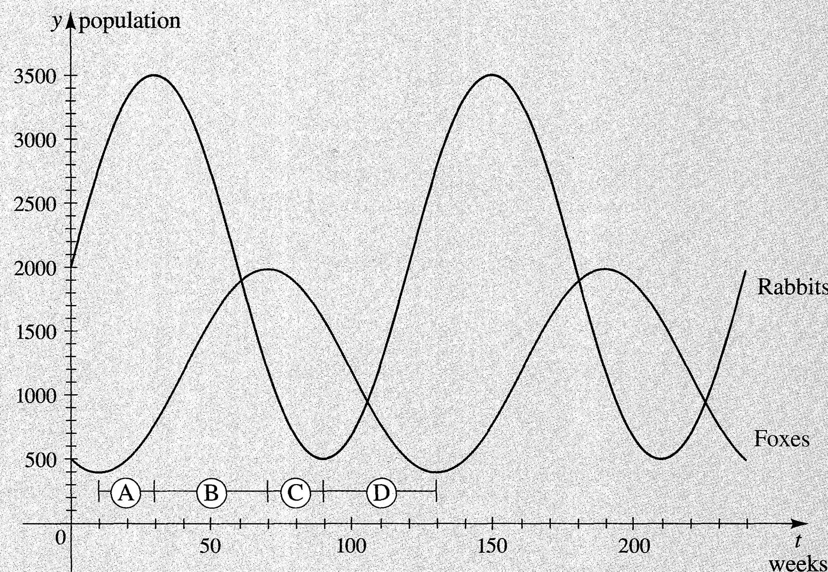

FIGURE 1

increases. By the time portrayed in part B, so many foxes are feeding on the rabbits that the rabbit population declines. In part C, the rabbit population has declined so much that there is not enough food for the foxes, so the fox population starts to decrease. In part D, so many foxes have died that the rabbits have few enemies and their population increases again. This takes us back where we started, and the cycle repeats over and over again.

The graphs in Figure 1 are sine curves that have been shifted upward, so they are graphs of functions of the form

$$y = a \sin k(t - b) + c$$

Here c is the amount by which the sine curve has been shifted vertically (see Section 4.5). Note that c is the average value of the function, halfway between the highest and lowest values on the graph. The amplitude $|a|$ is the amount by which the graph varies above and below the average value (see Figure 2).

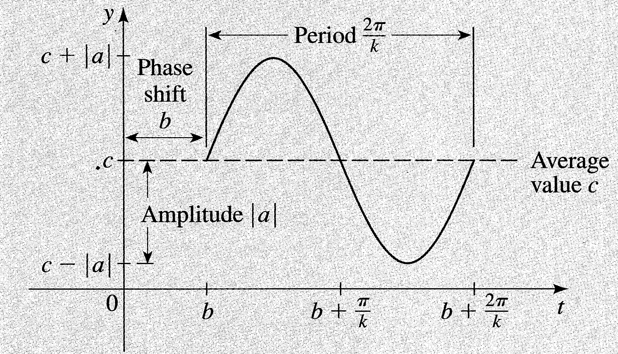

FIGURE 2
$y = a \sin k(t - b) + c$

1. Find functions of the form $y = a \sin k(t - b) + c$ that model the fox and rabbit populations graphed in Figure 1. Graph both functions on your calculator and compare to Figure 1 to verify that your functions are the right ones.

2. Add the fox and rabbit population functions to get a new function that models the total *mammal* population on this island. Graph this function on your calculator, and find its average value, amplitude, period, and phase shift. How are the average value and period of the mammal population function related to the average value and period of the fox and rabbit population functions?

3. A small lake on the island contains two species of fish: hake and redfish. The hake are predators that eat the redfish. The fish population in the lake varies periodically with period 180 days. The number of hake varies between 500 and 1500, and the number of redfish varies between 1000 and 3000. The hake reach their maximum population 30 days after the redfish have reached *their* maximum population in the cycle.

(a) Sketch a graph (like the one in Figure 1) that shows two complete periods of the population cycle for these species of fish. Assume that $t = 0$ corresponds to a time when the redfish population is at a maximum.

(b) Find cosine functions of the form $y = a \cos k(t - b) + c$ that model the hake and redfish populations in the lake.

4. In real life, most predator/prey populations do not behave as simply as the examples we have described here. In most cases, the populations of predator and prey oscillate, but the amplitude of the oscillations gets smaller and smaller, so that eventually both populations stabilize near a constant value.

(a) Sketch a graph that illustrates how the populations of predator and prey might behave in this case.

(b) What form of function could we use to model the populations in this case?