EXERCISES

1-30 ■ Calculate y'.

1.
$$y = (x^4 - 3x^2 + 5)^3$$
 2. $y = \cos(\tan x)$

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3.
$$y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$$

4.
$$y = \frac{3x-2}{\sqrt{2x+1}}$$

5.
$$y = 2x\sqrt{x^2 + 1}$$

6.
$$y = \frac{e^x}{1 + x^2}$$

7.
$$v = e^{\sin 2\theta}$$

8.
$$y = e^{-t}(t^2 - 2t + 2)$$

9.
$$y = \frac{t}{1 - t^2}$$

10.
$$y = \sin^{-1}(e^x)$$

11.
$$y = xe^{-1/x}$$

12.
$$y = x^r e^{sx}$$

$$13. \ xy^4 + x^2y = x + 3y$$

14.
$$y = \ln(\csc 5x)$$

$$\mathbf{15.} \ \ y = \frac{\sec 2\theta}{1 + \tan 2\theta}$$

16.
$$x^2 \cos y + \sin 2y = xy$$

17.
$$y = e^{cx}(c \sin x - \cos x)$$

18.
$$y = \ln(x^2 e^x)$$

19.
$$y = \log_5(1 + 2x)$$

20.
$$y = (\ln x)^{\cos x}$$

21.
$$y = \ln \sin x - \frac{1}{2} \sin^2 x$$

22.
$$y = \frac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$$

23.
$$y = x \tan^{-1}(4x)$$

24.
$$y = e^{\cos x} + \cos(e^x)$$

25.
$$y = \ln|\sec 5x + \tan 5x|$$

26.
$$v = 10^{\tan \pi \theta}$$

27.
$$y = \cot(3x^2 + 5)$$

28.
$$y = \ln \left| \frac{x^2 - 4}{2x + 5} \right|$$

29.
$$y = \sin(\tan\sqrt{1 + x^3})$$

30.
$$y = \arctan(\arcsin\sqrt{x})$$

31. If $f(x) = 1/(2x - 1)^5$, find f''(0).

32. Find
$$y''$$
 if $x^6 + y^6 = 1$.

33. If
$$f(x) = 2^x$$
, find $f^{(n)}(x)$.

34. Find an equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point (4, 1).

- **35.** (a) If $f(x) = x\sqrt{5-x}$, find f'(x).
 - (b) Find equations of the tangent lines to the curve $y = x\sqrt{5 - x}$ at the points (1, 2) and (4, 4).

M (c) Illustrate part (b) by graphing the curve and tangent lines on the same screen.

M (d) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

36. (a) If $f(x) = 4x - \tan x$, $-\pi/2 < x < \pi/2$, find f' and f''.

(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

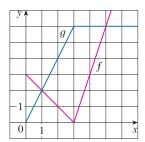
37. If $f(x) = xe^{\sin x}$, find f'(x). Graph f and f' on the same screen and comment.

38. (a) Graph the function $f(x) = x - 2 \sin x$ in the viewing rectangle [0, 8] by [-2, 8].

- (b) On which interval is the average rate of change larger: [1, 2] or [2, 3]?
- (c) At which value of x is the instantaneous rate of change larger: x = 2 or x = 5?
- (d) Check your visual estimates in part (c) by computing f'(x) and comparing the numerical values of f'(2)and f'(5).

39. Suppose that h(x) = f(x)g(x) and F(x) = f(g(x)), where f(2) = 3, g(2) = 5, g'(2) = 4, f'(2) = -2, and f'(5) = 11. Find (a) h'(2) and (b) F'(2).

40. If f and g are the functions whose graphs are shown, let P(x) = f(x)g(x), Q(x) = f(x)/g(x), and C(x) = f(g(x)).Find (a) P'(2), (b) Q'(2), and (c) C'(2).



41–48 Find f' in terms of g'.

41.
$$f(x) = x^2 g(x)$$

42.
$$f(x) = g(x^2)$$

43.
$$f(x) = [g(x)]^2$$

44.
$$f(x) = g(g(x))$$

45.
$$f(x) = g(e^x)$$

46.
$$f(x) = e^{g(x)}$$

47.
$$f(x) = \ln |g(x)|$$

48.
$$f(x) = g(\ln x)$$

49–50 Find h' in terms of f' and g'.

49.
$$h(x) = \frac{f(x)g(x)}{f(x) + g(x)}$$
 50. $h(x) = f(g(\sin 4x))$

50.
$$h(x) = f(g(\sin 4x))$$

51. At what point on the curve $y = [\ln(x+4)]^2$ is the tangent horizontal?

52. (a) Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line x - 4y = 1.

(b) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

53. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

54. (a) On what interval is the function $f(x) = (\ln x)/x$ increasing?

(b) On what interval is f concave upward?