9. There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.

10. There exists a function f such that $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.

11. If $f'(x)$ exists and is nonzero for all x, then $f(1) \neq f(0)$.

12. The most general antiderivative of $f(x) = x^{-2}$ is

$$F(x) = -\frac{1}{x} + C$$

13. $\lim_{x \to 0} \frac{x}{e^x} = 1$

EXERCISES

1–4 ■ Find the local and absolute extreme values of the function on the given interval.

1. $f(x) = 10 + 27x - x^3$, $[0, 4]$

2. $f(x) = x - \sqrt{x}$, $[0, 4]$

3. $f(x) = \dfrac{x}{x^2 + x + 1}$, $[-2, 0]$

4. $f(x) = x^2 e^{-x}$, $[0, 3]$

5–12 ■

(a) Find the vertical and horizontal asymptotes, if any.
(b) Find the intervals of increase or decrease.
(c) Find the local maximum and minimum values.
(d) Find the intervals of concavity and the inflection points.
(e) Use the information from parts (a)–(d) to sketch the graph of f. Check your work with a graphing device.

5. $f(x) = 2 - 2x - x^3$

6. $f(x) = x^4 + 4x^3$

7. $f(x) = x + \sqrt{1 - x}$

8. $f(x) = \dfrac{1}{1 - x^2}$

9. $y = \sin^2 x - 2\cos x$

10. $y = e^{2x - x^2}$

11. $y = e^x + e^{-3x}$

12. $y = \ln(x^2 - 1)$

13–16 ■ Produce graphs of f that reveal all the important aspects of the curve. Use graphs of f' and f'' to estimate the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points. In Exercise 13 use calculus to find these quantities exactly.

13. $f(x) = \dfrac{x^2 - 1}{x^3}$

14. $f(x) = \dfrac{\sqrt[3]{x}}{1 - x}$

15. $f(x) = 3x^6 - 5x^5 + x^4 - 5x^3 - 2x^2 + 2$

16. $f(x) = \sin x \cos^2 x$, $0 \leqslant x \leqslant 2\pi$

17. Graph $f(x) = e^{-1/x^2}$ in a viewing rectangle that shows all the main aspects of this function. Estimate the inflection points. Then use calculus to find them exactly.

CAS **18.** (a) Graph the function $f(x) = 1/(1 + e^{1/x})$.
(b) Explain the shape of the graph by computing the limits of $f(x)$ as x approaches ∞, $-\infty$, 0^+, and 0^-.
(c) Use the graph of f to estimate the coordinates of the inflection points.
(d) Use your CAS to compute and graph f''.
(e) Use the graph in part (d) to estimate the inflection points more accurately.

CAS **19.** If $f(x) = \arctan(\cos(3 \arcsin x))$, use the graphs of f, f', and f'' to estimate the x-coordinates of the maximum and minimum points and inflection points of f.

CAS **20.** If $f(x) = \ln(2x + x \sin x)$, use the graphs of f, f', and f'' to estimate the intervals of increase and the inflection points of f on the interval $(0, 15]$.

21. Investigate the family of functions $f(x) = \ln(\sin x + C)$. What features do the members of this family have in common? How do they differ? For which values of C is f continuous on $(-\infty, \infty)$? For which values of C does f have no graph at all? What happens as $C \to \infty$?

22. Investigate the family of functions $f(x) = cxe^{-cx^2}$. What happens to the maximum and minimum points and the inflection points as c changes? Illustrate your conclusions by graphing several members of the family.

23. For what values of the constants a and b is $(1, 6)$ a point of inflection of the curve $y = x^3 + ax^2 + bx + 1$?

24. Let $g(x) = f(x^2)$, where f is twice differentiable for all x, $f'(x) > 0$ for all $x \neq 0$, and f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$.
(a) At what numbers does g have an extreme value?
(b) Discuss the concavity of g.

25–32 ■ Evaluate the limit.

25. $\lim_{x \to \pi} \dfrac{\sin x}{x^2 - \pi^2}$

26. $\lim_{x \to 0} \dfrac{e^{ax} - e^{bx}}{x}$

27. $\lim_{x \to \infty} \dfrac{\ln(\ln x)}{\ln x}$

28. $\lim_{x \to 0} \dfrac{1 + \sin x - \cos x}{1 - \sin x - \cos x}$

29. $\lim_{x \to 0} \dfrac{\ln(1 - x) + x + \frac{1}{2}x^2}{x^3}$

30. $\lim_{x \to \pi/2} \left(\dfrac{\pi}{2} - x\right) \tan x$

31. $\lim_{x \to 0} (\csc^2 x - x^{-2})$

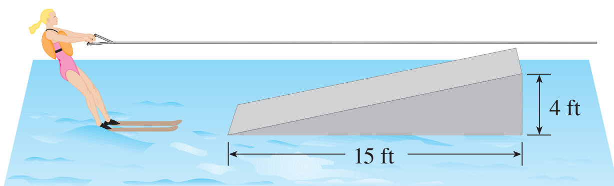
32. $\lim_{x \to 1} x^{1/(1-x)}$

33. The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the Sun is $\pi/6$?

34. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

35. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

36. A waterskier skis over the ramp shown in the figure at a speed of 30 ft/s. How fast is she rising as she leaves the ramp?



37. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

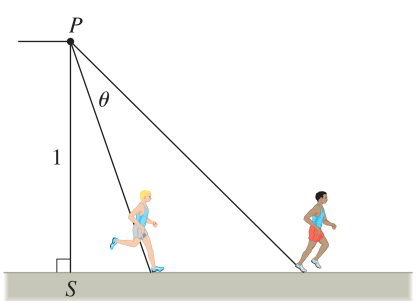
38. Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

39. Find the smallest possible area of an isosceles triangle that is circumscribed about a circle of radius r.

40. Find the volume of the largest circular cone that can be inscribed in a sphere of radius r.

41. In ΔABC, D lies on AB, $|CD| = 5$ cm, $|AD| = 4$ cm, $|BD| = 4$ cm, and $CD \perp AB$. Where should a point P be chosen on CD so that the sum $|PA| + |PB| + |PC|$ is a minimum? What if $|CD| = 2$ cm?

42. An observer stands at a point P, one unit away from a track. Two runners start at the point S in the figure and run along the track. One runner runs three times as fast as the other. Find the maximum value of the observer's angle of sight θ between the runners. [*Hint:* Maximize $\tan\theta$.]



43. The velocity of a wave of length L in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

where K and C are known positive constants. What is the length of the wave that gives the minimum velocity?

44. A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal?

45. A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at \$12, average attendance at a game has been 11,000. A market survey indicates that for each dollar the ticket price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price to maximize their revenue from ticket sales?

46. A manufacturer determines that the cost of making x units of a commodity is

$$C(x) = 1800 + 25x - 0.2x^2 + 0.001x^3$$

and the demand function is

$$p(x) = 48.2 - 0.03x$$

(a) Graph the cost and revenue functions and use the graphs to estimate the production level for maximum profit.
(b) Use calculus to find the production level for maximum profit.
(c) Estimate the production level that minimizes the average cost.

47. Use Newton's method to find the absolute minimum value of the function $f(x) = x^6 + 2x^2 - 8x + 3$ correct to six decimal places.

48. Use Newton's method to find all roots of the equation $6\cos x = x$ correct to six decimal places.

49–50 ■ Find the most general antiderivative of the function.

49. $f(x) = e^x - (2/\sqrt{x})$ **50.** $g(t) = (1 + t)/\sqrt{t}$

51–54 ■ Find $f(x)$.

51. $f'(x) = 2/(1 + x^2)$, $f(0) = -1$

52. $f'(x) = 1 + 2\sin x - \cos x$, $f(0) = 3$

53. $f''(x) = x^3 + x$, $f(0) = -1$, $f'(0) = 1$

54. $f''(x) = x^4 - 4x^2 + 3x - 2$, $f(0) = 0$, $f(1) = 1$