11. $\int_0^4 \frac{x}{x^2 - 1}\,dx = \frac{1}{2}\ln 15$

12. $\int_1^\infty \frac{1}{x^{\sqrt{2}}}\,dx$ is convergent.

13. $\int_0^2 (x - x^3)\,dx$ represents the area under the curve $y = x - x^3$ from 0 to 2.

14. All continuous functions have antiderivatives.

15. All continuous functions have derivatives.

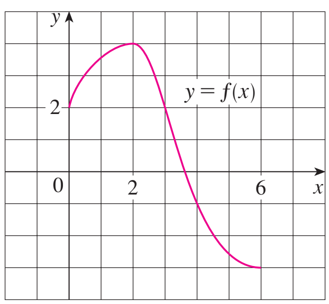
16. The Midpoint Rule is always more accurate than the Trapezoidal Rule.

17. If f is continuous, then $\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\,dx$.

18. If $f(x) \leqslant g(x)$ and $\int_0^\infty g(x)\,dx$ diverges, then $\int_0^\infty f(x)\,dx$ also diverges.

EXERCISES

1. Use the given graph of f to find the Riemann sum with six subintervals. Take the sample points to be (a) left endpoints and (b) midpoints. In each case draw a diagram and explain what the Riemann sum represents.



2. (a) Evaluate the Riemann sum for

$$f(x) = x^2 - x \qquad 0 \leqslant x \leqslant 2$$

with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

(b) Use the definition of a definite integral (with right endpoints) to calculate the value of the integral

$$\int_0^2 (x^2 - x)\,dx$$

(c) Use the Evaluation Theorem to check your answer to part (b).

(d) Draw a diagram to explain the geometric meaning of the integral in part (b).

3. Evaluate

$$\int_0^1 \left(x + \sqrt{1 - x^2}\right) dx$$

by interpreting it in terms of areas.

4. Express

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin x_i\,\Delta x$$

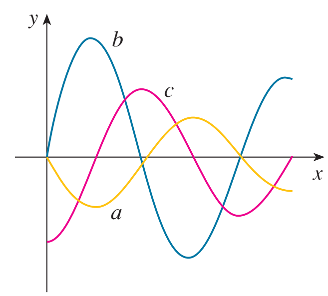
as a definite integral on the interval $[0, \pi]$ and then evaluate the integral.

5. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

CAS **6.** (a) Write $\int_0^2 e^{3x}\,dx$ as a limit of Riemann sums, taking the sample points to be right endpoints. Use a computer algebra system to evaluate the sum and to compute the limit.

(b) Use the Evaluation Theorem to check your answer to part (a).

7. The following figure shows the graphs of f, f', and $\int_0^x f(t)\,dt$. Identify each graph, and explain your choices.



8. Evaluate:

(a) $\int_0^1 \frac{d}{dx}\left(e^{\arctan x}\right) dx$

(b) $\frac{d}{dx}\int_0^1 e^{\arctan x}\,dx$

(c) $\frac{d}{dx}\int_0^x e^{\arctan t}\,dt$

9–34 ■ Evaluate the integral, if it exists.

9. $\int_1^2 (8x^3 + 3x^2)\,dx$

10. $\int_0^T (x^4 - 8x + 7)\,dx$

11. $\int_0^1 (1 - x^9)\,dx$

12. $\int_0^1 (1 - x)^9\,dx$

13. $\int_1^8 \sqrt[3]{x}\,(x - 1)\,dx$

14. $\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\,dx$

15. $\int_0^1 \frac{x}{x^2 + 1}\,dx$

16. $\int_0^1 \frac{1}{x^2 + 1}\,dx$

17. $\int_0^2 x^2(1 + 2x^3)^3\,dx$

18. $\int_0^4 x\sqrt{16 - 3x}\,dx$

19. $\int_0^1 e^{\pi t}\,dt$

20. $\int_1^2 x^3 \ln x\,dx$

21. $\int x \sec x \tan x\,dx$

22. $\int_1^2 \frac{1}{2-3x}\,dx$

23. $\int \frac{\cos(1/t)}{t^2}\,dt$

24. $\int \sin x \cos(\cos x)\,dx$

25. $\int \frac{6x+1}{3x+2}\,dx$

26. $\int x \cos 3x\,dx$

27. $\int x^2 e^{-x}\,dx$

28. $\int \sin^4\theta \cos^3\theta\,d\theta$

29. $\int \frac{dt}{t^2+6t+8}$

30. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

31. $\int_0^3 x^3\sqrt{9-x^2}\,dx$

32. $\int \tan^{-1}x\,dx$

33. $\int \frac{\sec\theta \tan\theta}{1+\sec\theta}\,d\theta$

34. $\int_{-1}^1 \frac{\sin x}{1+x^2}\,dx$

35–36 ■ Evaluate the indefinite integral. Illustrate and check that your answer is reasonable by graphing both the function and its antiderivative (take $C = 0$).

35. $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$

36. $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

37. Use a graph to give a rough estimate of the area of the region that lies under the curve $y = x\sqrt{x}$, $0 \leqslant x \leqslant 4$. Then find the exact area.

38. Graph the function $f(x) = \cos^2 x \sin^3 x$ and use the graph to guess the value of the integral $\int_0^{2\pi} f(x)\,dx$. Then evaluate the integral to confirm your guess.

39–42 ■ Find the derivative of the function.

39. $F(x) = \int_1^x \sqrt{1+t^4}\,dt$

40. $g(x) = \int_1^{\cos x} \sqrt[3]{1-t^2}\,dt$

41. $y = \int_{\sqrt{x}}^x \frac{e^t}{t}\,dt$

42. $y = \int_{2x}^{3x+1} \sin(t^4)\,dt$

43–46 ■ Use the Table of Integrals on the Reference Pages to evaluate the integral.

43. $\int e^x\sqrt{1-e^{2x}}\,dx$

44. $\int \csc^5 t\,dt$

45. $\int \sqrt{x^2+x+1}\,dx$

46. $\int \frac{\cot x}{\sqrt{1+2\sin x}}\,dx$

47–48 ■ Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule with $n = 10$ to approximate the given integral. Round your answers to six decimal places. Can you say whether your answers are underestimates or overestimates?

47. $\int_0^1 \sqrt{1+x^4}\,dx$

48. $\int_0^{\pi/2} \sqrt{\sin x}\,dx$

49. Estimate the errors involved in Exercise 47, parts (a) and (b). How large should n be in each case to guarantee an error of less than 0.00001?

50. Use Simpson's Rule with $n = 6$ to estimate the area under the curve $y = e^x/x$ from $x = 1$ to $x = 4$.

CAS **51.** (a) If $f(x) = \sin(\sin x)$, use a graph to find an upper bound for $|f^{(4)}(x)|$.
(b) Use Simpson's Rule with $n = 10$ to approximate $\int_0^\pi f(x)\,dx$ and use part (a) to estimate the error.
(c) How large should n be to guarantee that the size of the error in using S_n is less than 0.00001?

CAS **52.** (a) How would you evaluate $\int x^5 e^{-2x}\,dx$ by hand? (Don't actually carry out the integration.)
(b) How would you evaluate $\int x^5 e^{-2x}\,dx$ using tables? (Don't actually do it.)
(c) Use a CAS to evaluate $\int x^5 e^{-2x}\,dx$.
(d) Graph the integrand and the indefinite integral on the same screen.

53. Use Property 8 of integrals to estimate the value of $\int_1^3 \sqrt{x^2+3}\,dx$.

54. Use the properties of integrals to verify that

$$0 \leqslant \int_0^1 x^4 \cos x\,dx \leqslant 0.2$$

55–60 ■ Evaluate the integral or show that it is divergent.

55. $\int_1^\infty \frac{1}{(2x+1)^3}\,dx$

56. $\int_0^\infty \frac{\ln x}{x^4}\,dx$

57. $\int_{-\infty}^0 e^{-2x}\,dx$

58. $\int_0^1 \frac{1}{2-3x}\,dx$

59. $\int_1^e \frac{dx}{x\sqrt{\ln x}}$

60. $\int_2^6 \frac{y}{\sqrt{y-2}}\,dy$

61. Use the Comparison Theorem to determine whether the integral

$$\int_1^\infty \frac{x^3}{x^5+2}\,dx$$

is convergent or divergent.

62. For what values of a is $\int_0^\infty e^{ax}\cos x\,dx$ convergent? Use the Table of Integrals to evaluate the integral for those values of a.

63. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 5]$.