EXERCISES

1–2 ■ Find the area of the region bounded by the given curves.

1. $y = e^x - 1, \quad y = x^2 - x, \quad x = 1$

2. $x - 2y + 7 = 0, \quad y^2 - 6y - x = 0$

3. The curve traced out by a point at a distance 1 m from the center of a circle of radius 2 m as the circle rolls along the x-axis is called a *trochoid* and has parametric equations

$$x = 2\theta - \sin\theta \qquad y = 2 - \cos\theta$$

One arch of the trochoid is given by the parameter interval $0 \leqslant \theta \leqslant 2\pi$. Find the area under one arch of this trochoid.

4. Find the volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = e^{-2x}$, $y = 1 + x$, and $x = 1$.

5. Let $\mathcal{R}$ be the region bounded by the curves $y = \tan(x^2)$, $x = 1$, and $y = 0$. Use the Midpoint Rule with $n = 4$ to estimate the following.
(a) The area of $\mathcal{R}$
(b) The volume obtained by rotating $\mathcal{R}$ about the x-axis

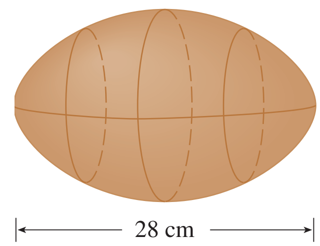
6. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities:
(a) The area of $\mathcal{R}$
(b) The volume obtained by rotating $\mathcal{R}$ about the x-axis
(c) The volume obtained by rotating $\mathcal{R}$ about the y-axis

7. Find the volumes of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the following lines:
(a) The x-axis (b) The y-axis (c) $y = 2$

8. Let $\mathcal{R}$ be the region bounded by the curves $y = 1 - x^2$ and $y = x^6 - x + 1$. Estimate the following quantities:
(a) The x-coordinates of the points of intersection of the curves
(b) The area of $\mathcal{R}$
(c) The volume generated when $\mathcal{R}$ is rotated about the x-axis
(d) The volume generated when $\mathcal{R}$ is rotated about the y-axis

9. Describe the solid whose volume is given by the integral.
(a) $\int_0^{\pi/2} 2\pi \cos^2 x \, dx$
(b) $\int_0^1 \pi\left[(2 - x^2)^2 - (2 - \sqrt{x})^2\right] dx$

10. Suppose you are asked to estimate the volume of a football. You measure and find that a football is 28 cm long. You use a piece of string and measure the circumference at its widest point to be 53 cm. The circumference 7 cm from each end is 45 cm. Use Simpson's Rule to make your estimate.



11. The base of a solid is a circular disk with radius 3. Find the volume of the solid if parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse lying along the base.

12. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side lying along the base.

13. The height of a monument is 20 m. A horizontal cross-section at a distance x meters from the top is an equilateral triangle with side $x/4$ meters. Find the volume of the monument.

14. (a) The base of a solid is a square with vertices located at $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$. Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.
(b) Show that by cutting the solid of part (a), we can rearrange it to form a cone. Thus compute its volume more simply.

15. Find the length of the curve with parametric equations $x = 3t^2$, $y = 2t^3$, $0 \leqslant t \leqslant 2$.

16. Use Simpson's Rule with $n = 10$ to estimate the length of the arc of the curve $y = 1/x^2$ from $(1, 1)$ to $\left(2, \frac{1}{4}\right)$.

17. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?

18. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

19. A tank full of water has the shape of a paraboloid of revolution as shown in the figure; that is, its shape is obtained by rotating a parabola about a vertical axis.