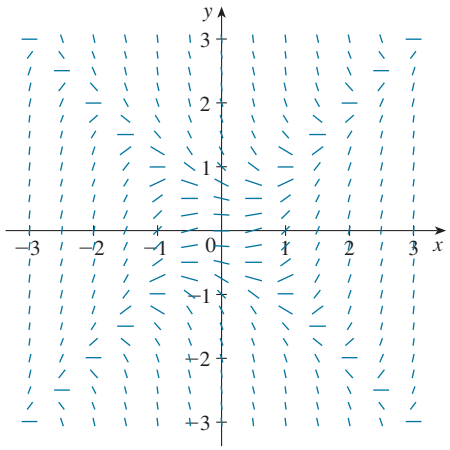
3. (a) A direction field for the differential equation $y' = x^2 - y^2$ is shown. Sketch the solution of the initial-value problem

$$y' = x^2 - y^2 \qquad y(0) = 1$$

Use your graph to estimate the value of $y(0.3)$.



(b) Use Euler's method with step size 0.1 to estimate $y(0.3)$ where $y(x)$ is the solution of the initial-value problem in part (a). Compare with your estimate from part (a).

(c) On what lines are the centers of the horizontal line segments of the direction field in part (a) located? What happens when a solution curve crosses these lines?

4. (a) Use Euler's method with step size 0.2 to estimate $y(0.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = 2xy^2 \qquad y(0) = 1$$

(b) Repeat part (a) with step size 0.1.

(c) Find the exact solution of the differential equation and compare the value at 0.4 with the approximations in parts (a) and (b).

5–6 ■ Solve the differential equation.

5. $(3y^2 + 2y)y' = x \cos x$ **6.** $\dfrac{dx}{dt} = 1 - t + x - tx$

7–8 ■ Solve the initial-value problem.

7. $xyy' = \ln x, \quad y(1) = 2$

8. $1 + x = 2xyy', \quad x > 0, \quad y(1) = -2$

9–10 ■ Find the orthogonal trajectories of the family of curves.

9. $kx^2 + y^2 = 1$ **10.** $y = \dfrac{k}{1 + x^2}$

11. A bacteria culture starts with 1000 bacteria and the growth rate is proportional to the number of bacteria. After 2 hours the population is 9000.

(a) Find an expression for the number of bacteria after t hours.

(b) Find the number of bacteria after 3 h.

(c) Find the rate of growth after 3 h.

(d) How long does it take for the number of bacteria to double?

12. An isotope of strontium, ^{90}Sr, has a half-life of 25 years.

(a) Find the mass of ^{90}Sr that remains from a sample of 18 mg after t years.

(b) How long would it take for the mass to decay to 2 mg?

13. Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Thus, $C'(t) = -kC(t)$, where k is a positive number called the *elimination constant* of the drug.

(a) If C_0 is the concentration at time $t = 0$, find the concentration at time t.

(b) If the body eliminates half the drug in 30 h, how long does it take to eliminate 90% of the drug?

14. (a) The population of the world was 5.28 billion in 1990 and 6.07 billion in 2000. Find an exponential model for these data and use the model to predict the world population in the year 2020.

(b) According to the model in part (a), when will the world population exceed 10 billion?

(c) Use the data in part (a) to find a logistic model for the population. Assume a carrying capacity of 100 billion. Then use the logistic model to predict the population in 2020. Compare with your prediction from the exponential model.

(d) According to the logistic model, when will the world population exceed 10 billion? Compare with your prediction in part (b).

15. The von Bertalanffy growth model is used to predict the length $L(t)$ of a fish over a period of time. If L_∞ is the largest length for a species, then the hypothesis is that the rate of growth in length is proportional to $L_\infty - L$, the length yet to be achieved.

(a) Formulate and solve a differential equation to find an expression for $L(t)$.

(b) For the North Sea haddock it has been determined that $L_\infty = 53$ cm, $L(0) = 10$ cm, and the constant of proportionality is 0.2. What does the expression for $L(t)$ become with these data?

16. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?