ANTON
10.3

3–14 Determine whether the series converges, and if so find its sum.

3. $\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$

4. $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$

5. $\sum_{k=1}^{\infty}(-1)^{k-1}\frac{7}{6^{k-1}}$

6. $\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^{k+1}$

7. $\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

8. $\sum_{k=1}^{\infty}\left(\frac{1}{2^k}-\frac{1}{2^{k+1}}\right)$

9. $\sum_{k=1}^{\infty}\frac{1}{9k^2+3k-2}$

10. $\sum_{k=2}^{\infty}\frac{1}{k^2-1}$

11. $\sum_{k=3}^{\infty}\frac{1}{k-2}$

12. $\sum_{k=5}^{\infty}\left(\frac{e}{\pi}\right)^{k-1}$

13. $\sum_{k=1}^{\infty}\frac{4^{k+2}}{7^{k-1}}$

14. $\sum_{k=1}^{\infty}5^{3k}7^{1-k}$

17–20 Express the repeating decimal as a fraction.

17. 0.4444...

18. 0.9999...

19. 5.373737...

20. 0.451141414...

21. Recall that a *terminating decimal* is a decimal whose digits are all 0 from some point on ($0.5 = 0.50000\ldots$, for example). Show that a decimal of the form $0.a_1a_2\ldots a_n9999\ldots$, where $a_n \neq 9$, can be expressed as a terminating decimal.

FOCUS ON CONCEPTS

22. The great Swiss mathematician Leonhard Euler (biography on p. 3) sometimes reached incorrect conclusions in his pioneering work on infinite series. For example, Euler deduced that

$$\tfrac{1}{2} = 1 - 1 + 1 - 1 + \cdots$$

and

$$-1 = 1 + 2 + 4 + 8 + \cdots$$

by substituting $x = -1$ and $x = 2$ in the formula

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

What was the problem with his reasoning?

23. A ball is dropped from a height of 10 m. Each time it strikes the ground it bounces vertically to a height that is $\frac{3}{4}$ of the preceding height. Find the total distance the ball will travel if it is assumed to bounce infinitely often.

24. The accompanying figure shows an "infinite staircase" constructed from cubes. Find the total volume of the staircase, given that the largest cube has a side of length 1 and each successive cube has a side whose length is half that of the preceding cube.

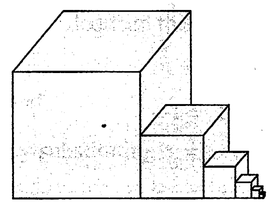

Figure Ex-24

25. In each part, find a closed form for the nth partial sum of the series, and determine whether the series converges. If so, find its sum.

(a) $\ln\frac{1}{2} + \ln\frac{2}{3} + \ln\frac{3}{4} + \cdots + \ln\frac{k}{k+1} + \cdots$

(b) $\ln\left(1-\frac{1}{4}\right) + \ln\left(1-\frac{1}{9}\right) + \ln\left(1-\frac{1}{16}\right) + \cdots + \ln\left(1-\frac{1}{(k+1)^2}\right) + \cdots$

26. Use geometric series to show that

(a) $\sum_{k=0}^{\infty}(-1)^k x^k = \frac{1}{1+x}$ if $-1 < x < 1$

(b) $\sum_{k=0}^{\infty}(x-3)^k = \frac{1}{4-x}$ if $2 < x < 4$

(c) $\sum_{k=0}^{\infty}(-1)^k x^{2k} = \frac{1}{1+x^2}$ if $-1 < x < 1$.

27. In each part, find all values of x for which the series converges, and find the sum of the series for those values of x.

(a) $x - x^3 + x^5 - x^7 + x^9 - \cdots$

(b) $\frac{1}{x^2} + \frac{2}{x^3} + \frac{4}{x^4} + \frac{8}{x^5} + \frac{16}{x^6} + \cdots$

(c) $e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + e^{-5x} + \cdots$

28. Show that for all real values of x

$$\sin x - \frac{1}{2}\sin^2 x + \frac{1}{4}\sin^3 x - \frac{1}{8}\sin^4 x + \cdots = \frac{2\sin x}{2+\sin x}$$

29. Let a_1 be any real number, and let $\{a_n\}$ be the sequence defined recursively by

$$a_{n+1} = \tfrac{1}{2}(a_n + 1)$$

Make a conjecture about the limit of the sequence, and confirm your conjecture by expressing a_n in terms of a_1 and taking the limit.

30. Show: $\sum_{k=1}^{\infty}\frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k^2+k}} = 1.$

31. Show: $\sum_{k=1}^{\infty}\left(\frac{1}{k}-\frac{1}{k+2}\right) = \frac{3}{2}.$

32. Show: $\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \cdots = \frac{3}{4}.$

33. Show: $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots = \frac{1}{2}.$