27. Let x represent the number of dollars that the ticket prices are lowered. So the new ticket price is $10 - x$, and the number of tickets sold is $n = 27000 + 3000x$. Thus the revenue is $R(x) = (10 - x)(27000 + 3000x) = 270000 + 3000x - 3000x^2 = -3000(x^2 - x) + 270000$ $= -3000\left(x^2 - x + \frac{1}{4}\right) + 270000 + 750 = -3000\left(x - \frac{1}{2}\right)^2 + 270750$. The revenue is maximized when $x = \frac{1}{2}$, and so the price should be set at $10 - \frac{1}{2} = \$9.50$.

29. Let h be the height in feet of the straight portion of the window. The circumference of the semicircle is $C = \frac{1}{2}\pi x$. Since the perimeter of the window is 30 feet, we have $x + 2h + \frac{1}{2}\pi x = 30$. Solving for h, we get $2h = 30 - x - \frac{1}{2}\pi x \quad \Leftrightarrow \quad h = 15 - \frac{1}{2}x - \frac{1}{4}\pi x$. The area of the window is $A(x) = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = x\left(15 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{1}{8}\pi x^2 = 15x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x^2$ $= 15x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2 = 15x - \frac{1}{8}(\pi + 4)x^2 = -\frac{1}{8}(\pi + 4)\left[x^2 - \frac{120}{\pi+4}x\right]$ $= -\frac{1}{8}(\pi + 4)\left[x^2 - \frac{120}{\pi+4}x + \left(\frac{60}{\pi+4}\right)^2\right] + \frac{450}{\pi+4} = -\frac{1}{8}(\pi + 4)\left(x - \frac{60}{\pi+4}\right)^2 + \frac{450}{\pi+4}$. The area is maximized when $x = \frac{60}{\pi+4} \approx 8.40$, and hence $h \approx 15 - \frac{1}{2}(8.40) - \frac{1}{4}\pi(8.40) \approx 4.20$.