24. Let w be the width of the rectangular area (in feet) and l be the length of the field (in feet). Since the farmer has 750 feet of fencing, we must have $5w + 2l = 750 \quad \Leftrightarrow \quad 2l = 750 - 5w \quad \Leftrightarrow$ $l = \frac{5}{2}(150 - w)$. The area of the four pens is $A(w) = l \cdot w = \frac{5}{2}w(150 - w) = -\frac{5}{2}(w^2 - 150w)$

$= -\frac{5}{2}(w^2 - 150w + 75^2) + \left(\frac{5}{2}\right) \cdot 75^2 = -\frac{5}{2}(w - 75)^2 + 14062.5$. Therefore, the largest possible area of the four pens is 14,062.5 square feet.

26. Let x be the length of wire in cm that is bent into a square. So $10 - x$ is the length of wire in cm that is bent into the second square. The width of each square is $\dfrac{x}{4}$ and $\dfrac{10 - x}{4}$, and the area of each square is $\left(\dfrac{x}{4}\right)^2 = \dfrac{x^2}{16}$ and $\left(\dfrac{10 - x}{4}\right)^2 = \dfrac{100 - 20x + x^2}{16}$. Thus the sum of the areas is

$$A(x) = \frac{x^2}{16} + \frac{100 - 20x + x^2}{16} = \frac{100 - 20x + 2x^2}{16} = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4}$$
$$= \tfrac{1}{8}(x^2 - 10x) + \tfrac{25}{4} = \tfrac{1}{8}(x^2 - 10x + 25) + \tfrac{25}{4} - \tfrac{25}{8} = \tfrac{1}{8}(x - 5)^2 + \tfrac{25}{8}.$$

So the minimum area is $\frac{25}{8}$ cm^2 when each piece is 5 cm long.

28. Let x be the number of one dollar increases in the price of a necklace. So the selling price will be $10 + x$ dollars, and the number of necklaces sold will be $20 - 2x$. The revenue from the sales will be $(10 + x)(20 - 2x)$, and the cost will be $6(20 - 2x)$. The profits will be

$P(x) = (10 + x)(20 - 2x) - 6(20 - 2x) = (4 + x)(20 - 2x) = 80 + 12x - 2x^2$

$= -2(x^2 - 6x) + 80 = -2(x^2 - 6x + 9) + 80 + 18 = -2(x - 3)^2 + 98$. Thus the profit would be maximized at \$98 when $x = 3$. So he should set the selling price at $10 + x = \$13$.