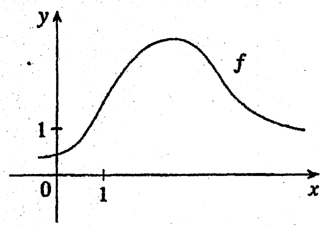
STEWART 8.7

1. If $f(x) = \sum_{n=0}^{\infty} b_n(x-5)^n$ for all x, write a formula for b_8.

2. (a) The graph of f is shown. Explain why the series

$$1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + \cdots$$

is *not* the Taylor series of f centered at 1.



(b) Explain why the series

$$2.8 + 0.5(x-2) + 1.5(x-2)^2 - 0.1(x-2)^3 + \cdots$$

is *not* the Taylor series of f centered at 2.

3–6 ■ Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.] Also find the associated radius of convergence.

3. $f(x) = \cos x$
4. $f(x) = \sin 2x$
5. $f(x) = (1+x)^{-3}$
6. $f(x) = \ln(1+x)$

7–14 ■ Find the Taylor series for $f(x)$ centered at the given value of a. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

7. $f(x) = 1 + x + x^2, \quad a = 2$
8. $f(x) = x^3, \quad a = -1$
9. $f(x) = e^x, \quad a = 3$
10. $f(x) = \ln x, \quad a = 2$
11. $f(x) = 1/x, \quad a = 1$
12. $f(x) = \sqrt{x}, \quad a = 4$
13. $f(x) = \sin x, \quad a = \pi/4$
14. $f(x) = \cos x, \quad a = -\pi/4$

15. Prove that the series obtained in Exercise 3 represents $\cos x$ for all x.

16. Prove that the series obtained in Exercise 13 represents $\sin x$ for all x.

17–24 ■ Use a Maclaurin series derived in this section to obtain the Maclaurin series for the given function.

17. $f(x) = \cos \pi x$
18. $f(x) = e^{-x/2}$
19. $f(x) = x \tan^{-1} x$
20. $f(x) = \sin(x^4)$
21. $f(x) = x^2 e^{-x}$
22. $f(x) = x \cos 2x$
23. $f(x) = \sin^2 x$ [*Hint:* Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.]

25–28 ■ Find the Maclaurin series of f (by any method) and its radius of convergence. Graph f and its first few Taylor polynomials on the same screen. What do you notice about the relationship between these polynomials and f?

25. $f(x) = \sqrt{1+x}$
26. $f(x) = 1/\sqrt{1+2x}$
27. $f(x) = \cos(x^2)$
28. $f(x) = 2^x$

29. Use the Maclaurin series for e^x to calculate $e^{-0.2}$ correct to five decimal places.

30. Use the Maclaurin series for $\sin x$ to compute $\sin 3°$ correct to five decimal places.

31–34 ■ Evaluate the indefinite integral as an infinite series.

31. $\int \sin(x^2)\,dx$
32. $\int \frac{\sin x}{x}\,dx$
33. $\int \sqrt{x^3+1}\,dx$
34. $\int e^{x^3}\,dx$

35–38 ■ Use series to approximate the definite integral to within the indicated accuracy.

35. $\int_0^1 \sin(x^2)\,dx$ (three decimal places)

36. $\int_0^{0.5} \cos(x^2)\,dx$ (three decimal places)

37. $\int_0^{0.1} \frac{dx}{\sqrt{1+x^3}}$ ($|\text{error}| < 10^{-8}$)

38. $\int_0^{0.5} x^2 e^{-x^2}\,dx$ ($|\text{error}| < 0.001$)

39–41 ■ Use series to evaluate the limit.

39. $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$
40. $\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$
41. $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

42. Use the series in Example 10(b) to evaluate

$$\lim_{x \to 0} \frac{\tan x - x}{x^3}$$

We found this limit in Example 4 in Section 4.5 using l'Hospital's Rule three times. Which method do you prefer?

43–46 ■ Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for each function.

43. $y = e^{-x^2} \cos x$
44. $y = \sec x$
45. $y = \frac{\ln(1-x)}{e^x}$
46. $y = e^x \ln(1-x)$

47–52 ■ Find the sum of the series.

47. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$
48. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$
49. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$
50. $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$
51. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$
52. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$