35. Using our series from Exercise 31, we get

$$\int_0^1 \sin(x^2)\,dx = \sum_{n=0}^{\infty}\left[\frac{(-1)^n\,x^{4n+3}}{(4n+3)(2n+1)!}\right]_0^1 = \sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+3)(2n+1)!}$$ and

$|c_3| = \dfrac{1}{75{,}600} < 0.000014$, so by the Alternating Series Estimation Theorem, we have

$$\int_0^1 \sin(x^2)\,dx \approx \sum_{n=0}^{2}\frac{(-1)^n}{(4n+3)(2n+1)!} = \frac{1}{3} - \frac{1}{42} + \frac{1}{1320} \approx 0.310$$ (correct to three decimal places).

36. $\cos(x^2) = \sum_{n=0}^{\infty}\dfrac{(-1)^n\,(x^2)^{2n}}{(2n)!}$, so

$$\int_0^{0.5}\cos(x^2)\,dx = \int_0^{0.5}\sum_{n=0}^{\infty}\frac{(-1)^n\,x^{4n}}{(2n)!}\,dx = \sum_{n=0}^{\infty}\left[\frac{(-1)^n\,x^{4n+1}}{(4n+1)(2n)!}\right]_0^{0.5} = 0.5 - \frac{(0.5)^5}{5\cdot 2!} + \frac{(0.5)^9}{9\cdot 4!} - \cdots,$$ but

$\dfrac{(0.5)^9}{9\cdot 4!} \approx 0.000009$, so by the Alternating Series Estimation Theorem, $\int_0^{0.5}\cos(x^2)\,dx \approx 0.5 - \dfrac{(0.5)^5}{5\cdot 2!} \approx 0.497$ (correct to three decimal places).

38. $$\int_0^{0.5} x^2 e^{-x^2}\,dx = \int_0^{0.5}\sum_{n=0}^{\infty}\frac{(-1)^n\,x^{2n+2}}{n!}\,dx = \sum_{n=0}^{\infty}\left[\frac{(-1)^n\,x^{2n+3}}{n!(2n+3)}\right]_0^{1/2} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!(2n+3)2^{2n+3}}$$ and since

$c_2 = \dfrac{1}{1792} < 0.001$ we use $\sum_{n=0}^{1}\dfrac{(-1)^n}{n!(2n+3)2^{2n+3}} = \dfrac{1}{24} - \dfrac{1}{160} \approx 0.0354$.