40. $$\lim_{x\to0}\frac{1-\cos x}{1+x-e^x} = \lim_{x\to0}\frac{1-\left(1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots\right)}{1+x-\left(1+x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\frac{1}{5!}x^5+\frac{1}{6!}x^6+\cdots\right)}$$

$$= \lim_{x\to0}\frac{\frac{1}{2!}x^2-\frac{1}{4!}x^4+\frac{1}{6!}x^6-\cdots}{-\frac{1}{2!}x^2-\frac{1}{3!}x^3-\frac{1}{4!}x^4-\frac{1}{5!}x^5-\frac{1}{6!}x^6-\cdots}$$

$$= \lim_{x\to0}\frac{\frac{1}{2!}-\frac{1}{4!}x^2+\frac{1}{6!}x^4-\cdots}{-\frac{1}{2!}-\frac{1}{3!}x-\frac{1}{4!}x^2-\frac{1}{5!}x^3-\frac{1}{6!}x^4-\cdots} = \frac{\frac{1}{2}-0}{-\frac{1}{2}-0} = -1$$

since power series are continuous functions.

41. $$\lim_{x\to0}\frac{\sin x - x + \frac{1}{6}x^3}{x^5} = \lim_{x\to0}\frac{\left(x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\cdots\right)-x+\frac{1}{6}x^3}{x^5}$$

$$= \lim_{x\to0}\frac{\frac{1}{5!}x^5-\frac{1}{7!}x^7+\cdots}{x^5} = \lim_{x\to0}\left(\frac{1}{5!}-\frac{x^2}{7!}+\frac{x^4}{9!}-\cdots\right) = \frac{1}{5!} = \frac{1}{120}$$

since power series are continuous functions.

42. $$\lim_{x\to0}\frac{\tan x - x}{x^3} = \lim_{x\to0}\frac{\left(x+\frac{1}{3}x^3+\frac{2}{15}x^5+\cdots\right)-x}{x^3} = \lim_{x\to0}\frac{\frac{1}{3}x^3+\frac{2}{15}x^5+\cdots}{x^3} = \lim_{x\to0}\left(\frac{1}{3}+\frac{2}{15}x^2+\cdots\right) = \frac{1}{3}$$

since power series are continuous functions.

43. As in Example 8(a), we have $e^{-x^2} = 1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots$ and we know that $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$ from Equation 16. Therefore, $e^{-x^2}\cos x = \left(1-x^2+\frac{1}{2}x^4-\cdots\right)\left(1-\frac{1}{2}x^2+\frac{1}{24}x^4-\cdots\right)$. Writing only the terms with degree ≤ 4, we get $e^{-x^2}\cos x = 1-\frac{1}{2}x^2+\frac{1}{24}x^4-x^2+\frac{1}{2}x^4+\frac{1}{2}x^4+\cdots = 1-\frac{3}{2}x^2+\frac{25}{24}x^4+\cdots$.

44.

$$\begin{array}{r|l}
 & 1+\frac{1}{2}x^2+\frac{5}{24}x^4+\cdots \\ \hline
1-\frac{1}{2}x^2+\frac{1}{24}x^4-\cdots & 1 \\
 & \underline{1-\frac{1}{2}x^2+\frac{1}{24}x^4-\cdots} \\
 & \frac{1}{2}x^2-\frac{1}{24}x^4+\cdots \\
 & \underline{\frac{1}{2}x^2-\frac{1}{4}x^4+\cdots} \\
 & \frac{5}{24}x^4+\cdots \\
 & \underline{\frac{5}{24}x^4+\cdots} \\
 & \cdots
\end{array}$$

$\sec x = \dfrac{1}{\cos x} = \dfrac{1}{1-\frac{1}{2}x^2+\frac{1}{24}x^4-\cdots}$.

From the long division above,

$\sec x = 1+\frac{1}{2}x^2+\frac{5}{24}x^4+\cdots$.

45.

$$\begin{array}{r|l}
 & -x+\frac{1}{2}x^2-\frac{1}{3}x^3+\cdots \\ \hline
1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\cdots & -x-\frac{1}{2}x^2-\frac{1}{3}x^3-\cdots \\
 & \underline{-x-x^2-\frac{1}{2}x^3-\cdots} \\
 & \frac{1}{2}x^2+\frac{1}{6}x^3-\cdots \\
 & \underline{\frac{1}{2}x^2+\frac{1}{2}x^3+\cdots} \\
 & -\frac{1}{3}x^3+\cdots \\
 & \underline{-\frac{1}{3}x^3+\cdots} \\
 & \cdots
\end{array}$$

From Example 6 in Section 8.6, we have

$\ln(1-x) = -x-\frac{1}{2}x^2-\frac{1}{3}x^3-\cdots$, $|x|<1$.

Therefore,

$y = \dfrac{\ln(1-x)}{e^x} = \dfrac{-x-\frac{1}{2}x^2-\frac{1}{3}x^3-\cdots}{1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\cdots}$.

So by the long division above,

$\dfrac{\ln(1-x)}{e^x} = -x+\dfrac{x^2}{2}-\dfrac{x^3}{3}+\cdots$, $|x|<1$.