

71. $\int \frac{x+3}{\sqrt{(x+1)^2+1}} dx$, let $u = x+1$,

$$\text{let } x+1 = \tan \theta,$$

$$\begin{aligned}\int (\tan \theta + 2) \sec \theta d\theta &= \int \sec \theta \tan \theta d\theta + 2 \int \sec \theta d\theta = \sec \theta + 2 \ln |\sec \theta + \tan \theta| + C \\ &= \sqrt{x^2 + 2x + 2} + 2 \ln(\sqrt{x^2 + 2x + 2} + x + 1) + C.\end{aligned}$$

72. Let $x = \tan \theta$ to get $\int \frac{1}{x^3 - x^2} dx$.

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}; \quad A = -1, B = -1, C = 1 \text{ so}$$

$$-\int \frac{1}{x} dx - \int \frac{1}{x^2} dx + \int \frac{1}{x-1} dx = -\ln|x| + \frac{1}{x} + \ln|x-1| + C$$

$$= \frac{1}{x} + \ln \left| \frac{x-1}{x} \right| + C = \cot \theta + \ln \left| \frac{\tan \theta - 1}{\tan \theta} \right| + C = \cot \theta + \ln |1 - \cot \theta| + C$$