34. $\sqrt{x}+\sqrt{y}=3 \quad\Rightarrow\quad \dfrac{1}{2\sqrt{x}}+\dfrac{1}{2\sqrt{y}}\,y'=0 \quad\Rightarrow\quad \dfrac{1}{\sqrt{y}}\,y'=-\dfrac{1}{\sqrt{x}} \quad\Rightarrow\quad y'=-\dfrac{\sqrt{y}}{\sqrt{x}}.$

At $(4,1)$, $y'=-\frac{1}{2}$, so the tangent is $y-1=-\frac{1}{2}(x-4)$ or $y=-\frac{1}{2}x+3$.

51. $y=[\ln(x+4)]^2 \quad\Rightarrow\quad y'=2[\ln(x+4)]^1\cdot\dfrac{1}{x+4}\cdot 1=2\,\dfrac{\ln(x+4)}{x+4}$ and $y'=0 \quad\Leftrightarrow\quad \ln(x+4)=0 \quad\Leftrightarrow$ $x+4=e^0 \quad\Rightarrow\quad x+4=1 \quad\Leftrightarrow\quad x=-3$, so the tangent is horizontal at the point $(-3,0)$.

52. (a) The line $x-4y=1$ has slope $\frac{1}{4}$. A tangent to $y=e^x$ has slope $\frac{1}{4}$ when $y'=e^x=\frac{1}{4} \quad\Rightarrow$ $x=\ln\frac{1}{4}=-\ln 4$. Since $y=e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4,\frac{1}{4}\right)$. Thus, an equation of the tangent line is $y-\frac{1}{4}=\frac{1}{4}(x+\ln 4)$ or $y=\frac{1}{4}x+\frac{1}{4}(\ln 4+1)$.

(b) The slope of the tangent at the point (a,e^a) is $\dfrac{d}{dx}e^x\Big|_{x=a}=e^a$. Thus, an equation of the tangent line is $y-e^a=e^a(x-a)$. We substitute $x=0$, $y=0$ into this equation, since we want the line to pass through the origin: $0-e^a=e^a(0-a) \quad\Leftrightarrow\quad -e^a=e^a(-a) \quad\Leftrightarrow\quad a=1$. So an equation of the tangent line at the point $(a,e^a)=(1,e)$ is $y-e=e(x-1)$ or $y=ex$.

53. $x^2+2y^2=1 \quad\Rightarrow\quad 2x+4yy'=0 \quad\Rightarrow\quad y'=-x/(2y)=1 \quad\Leftrightarrow\quad x=-2y$. Since the points lie on the ellipse, we have $(-2y)^2+2y^2=1 \quad\Rightarrow\quad 6y^2=1 \quad\Rightarrow\quad y=\pm\frac{1}{\sqrt{6}}$. The points are $\left(-\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$ and $\left(\frac{2}{\sqrt{6}},-\frac{1}{\sqrt{6}}\right)$.