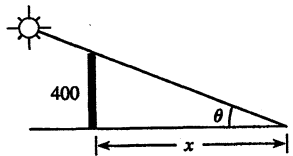


33. We are given $d\theta/dt = -0.25$ rad/h.

$$x = 400 \cot \theta \Rightarrow \frac{dx}{dt} = -400 \csc^2 \theta \frac{d\theta}{dt}. \text{ When } \theta = \frac{\pi}{6},$$

$$\frac{dx}{dt} = -400(2)^2(-0.25) = 400 \text{ ft/h.}$$

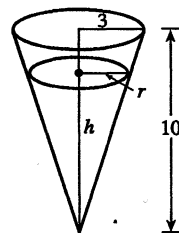


34. Given $dV/dt = 2$, find dh/dt when $h = 5$. $V = \frac{1}{3}\pi r^2 h$ and, from similar

$$\text{triangles, } \frac{r}{h} = \frac{3}{10} \Rightarrow V = \frac{\pi}{3} \left(\frac{3h}{10}\right)^2 h = \frac{3\pi}{100} h^3, \text{ so}$$

$$2 = \frac{dV}{dt} = \frac{9\pi}{100} h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{200}{9\pi h^2} = \frac{200}{9\pi(5)^2} = \frac{8}{9\pi} \text{ cm/s}$$

when $h = 5$.

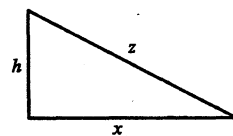


35. Given $dh/dt = 5$ and $dx/dt = 15$, find dz/dt . $z^2 = x^2 + h^2 \Rightarrow$

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2h \frac{dh}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z}(15x + 5h). \text{ When } t = 3,$$

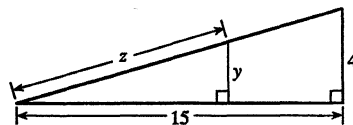
$$h = 45 + 3(5) = 60 \text{ and } x = 15(3) = 45 \Rightarrow z = 75, \text{ so}$$

$$\frac{dz}{dt} = \frac{1}{75}[15(45) + 5(60)] = 13 \text{ ft/s.}$$



36. We are given $dz/dt = 30$ ft/s. By similar triangles, $\frac{y}{z} = \frac{4}{\sqrt{241}}$

$$\Rightarrow y = \frac{4}{\sqrt{241}}z, \text{ so } \frac{dy}{dt} = \frac{4}{\sqrt{241}} \frac{dz}{dt} = \frac{120}{\sqrt{241}} \approx 7.7 \text{ ft/s.}$$



37. Call the two integers x and y . Then $x + 4y = 1000$, so $x = 1000 - 4y$. Their product is $P = xy = (1000 - 4y)y$, so our problem is to maximize the function $P(y) = 1000y - 4y^2$, where $0 < y < 250$ and y is an integer. $P'(y) = 1000 - 8y$, so $P'(y) = 0 \Leftrightarrow y = 125$. $P''(y) = -8 < 0$, so $P(125) = 62,500$ is an absolute maximum. Since the optimal y turned out to be an integer, we have found the desired pair of numbers, namely $x = 1000 - 4(125) = 500$ and $y = 125$.

45. Let x denote the number of \$1 decreases in ticket price. Then the ticket price is $\$12 - \$1(x)$, and the average attendance is $11,000 + 1000(x)$. Now the revenue per game is

$$\begin{aligned} R(x) &= (\text{price per person}) \times (\text{number of people per game}) \\ &= (12 - x)(11,000 + 1000x) = -1000x^2 + 1000x + 132,000 \end{aligned}$$

for $0 \leq x \leq 4$ (since the seating capacity is 15,000) $\Rightarrow R'(x) = -2000x + 1000 = 0 \Leftrightarrow x = 0.5$. This is a maximum since $R''(x) = -2000 < 0$ for all x . Now we must check the value of

$R(x) = (12 - x)(11,000 + 1000x)$ at $x = 0.5$ and at the endpoints of the domain to see which value of x gives the maximum value of R . $R(0) = (12)(11,000) = 132,000$, $R(0.5) = (11.5)(11,500) = 132,250$, and $R(4) = (8)(15,000) = 120,000$. Thus, the maximum revenue of \$132,250 per game occurs when the average attendance is 11,500 and the ticket price is \$11.50.