13. $\int_1^8 \sqrt[3]{x}\,(x-1)\,dx = \int_1^8 \left(x^{4/3} - x^{1/3}\right) dx = \left[\frac{3}{7}x^{7/3} - \frac{3}{4}x^{4/3}\right]_1^8 = \left(\frac{3}{7}\cdot 128 - \frac{3}{4}\cdot 16\right) - \left(\frac{3}{7} - \frac{3}{4}\right) = \frac{1209}{28}$

14. $\displaystyle\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\,dx = \int_1^4 \left(x^{3/2} - x^{1/2} + x^{-1/2}\right) dx = \left[\tfrac{2}{5}x^{5/2} - \tfrac{2}{3}x^{3/2} + 2x^{1/2}\right]_1^4$

$$= \left(\tfrac{2}{5}\cdot 32 - \tfrac{2}{3}\cdot 8 + 4\right) - \left(\tfrac{2}{5} - \tfrac{2}{3} + 2\right) = \tfrac{146}{15}$$

15. $u = x^2 + 1$, $du = 2x\,dx$, so $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx = \int_1^2 \frac{1}{u}\left(\frac{1}{2}\,du\right) = \frac{1}{2}\left[\ln u\right]_1^2 = \frac{1}{2}\ln 2.$

16. $\displaystyle\int_0^1 \frac{1}{x^2+1}\,dx = \left[\tan^{-1} x\right]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \tfrac{\pi}{4} - 0 = \tfrac{\pi}{4}$

17. Let $u = 1 + 2x^3$. Then $du = 6x^2\,dx$, so
$\int_0^2 x^2\left(1+2x^3\right)^3 dx = \int_1^{17} u^3\left(\frac{1}{6}\,du\right) = \left[\frac{1}{24}u^4\right]_1^{17} = \frac{1}{24}\left(17^4 - 1\right) = 3480.$

18. Let $u = 16 - 3x$. Then $x = \frac{1}{3}(16 - u)$, $dx = -\frac{1}{3}\,du$, so

$$\int_0^4 x\sqrt{16-3x}\,dx = \int_{16}^4 u^{1/2}\left(\frac{16-u}{3}\right)\left(-\frac{1}{3}\,du\right) = \frac{1}{9}\int_4^{16}\left(16u^{1/2} - u^{3/2}\right) du$$

$$= \tfrac{1}{9}\left[16\cdot\tfrac{2}{3}u^{3/2} - \tfrac{2}{5}u^{5/2}\right]_4^{16} = \tfrac{1}{9}\left[\tfrac{32}{3}\cdot 64 - \tfrac{2}{5}\cdot 1024 - \tfrac{32}{3}\cdot 8 + \tfrac{2}{5}\cdot 32\right] = \tfrac{3008}{135}$$

19. $\int_0^1 e^{\pi t}\,dt = \left[\frac{1}{\pi}e^{\pi t}\right]_0^1 = \frac{1}{\pi}\left(e^{\pi} - 1\right)$

20. Integrate by parts with $u = \ln x$, $dv = x^3\,dx \quad\Rightarrow\quad du = dx/x$, $v = x^4/4$:
$\int_1^2 x^3 \ln x\,dx = \left[\frac{1}{4}x^4 \ln x\right]_1^2 - \frac{1}{4}\int_1^2 x^3\,dx = 4\ln 2 - \frac{1}{16}\left[x^4\right]_1^2 = 4\ln 2 - \frac{15}{16}.$

21. Integrate by parts with $u = x$, $dv = \sec x \tan x\,dx \quad\Rightarrow\quad du = dx$, $v = \sec x$:
$\int x\sec x\,\tan x\,dx = x\sec x - \int \sec x\,dx \overset{14}{=} x\sec x - \ln|\sec x + \tan x| + C.$

22. Let $u = 2 - 3x$. Then $du = -3\,dx$, so $\displaystyle\int_1^2 \frac{1}{2-3x}\,dx = -\frac{1}{3}\int_{-1}^{-4}\frac{du}{u} = \left[-\frac{\ln|u|}{3}\right]_{-1}^{-4} = -\frac{\ln 4}{3}.$

23. Let $u = \dfrac{1}{t}$. Then $du = -\dfrac{1}{t^2}\,dt$, so $\displaystyle\int \frac{\cos(1/t)}{t^2}\,dt = \int \cos u\,(-du) = -\sin u + C = -\sin\left(\frac{1}{t}\right) + C.$

24. Let $u = \cos x$. Then $du = -\sin x\,dx$, so
$\int \sin x\,\cos(\cos x)\,dx = -\int \cos u\,du = -\sin u + C = -\sin(\cos x) + C.$

25. Since the degree of the numerator is equal to the degree of the denominator, we first change the form of the integrand by using long division.

$$\int \frac{6x+1}{3x+2}\,dx = \int\left(2 - \frac{3}{3x+2}\right) dx = 2x - 3\cdot\tfrac{1}{3}\ln|3x+2| + C = 2x - \ln|3x+2| + C$$

26. Let $u = x$, $dv = \cos 3x\,dx \quad\Rightarrow\quad du = dx$, $v = \frac{1}{3}\sin 3x$. Then
$\int x\cos 3x\,dx = \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx = \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C.$