55. $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty}\int_1^t \tfrac{1}{2}(2x+1)^{-3}\,2\,dx$

$$= \lim_{t\to\infty}\left[-\frac{1}{4(2x+1)^2}\right]_1^t = -\frac{1}{4}\lim_{t\to\infty}\left[\frac{1}{(2t+1)^2}-\frac{1}{9}\right] = -\frac{1}{4}\left(0-\frac{1}{9}\right) = \frac{1}{36}$$

56. $\displaystyle I = \int_0^\infty \frac{\ln x}{x^4}\,dx = \int_0^1 \frac{\ln x}{x^4}\,dx + \int_1^\infty \frac{\ln x}{x^4}\,dx = I_1 + I_2$. Integrate by parts with $u = \ln x$, $dv = dx/x^4$ $\Rightarrow$ $du = dx/x$, $v = -1/\left(3x^3\right)$:

$$\int \frac{\ln x}{x^4}\,dx = -\frac{\ln x}{3x^3} + \frac{1}{3}\int \frac{1}{x^4}\,dx = -\frac{\ln x}{3x^3} + \frac{1}{3}\left(-\frac{1}{3x^3}\right) + C = -\frac{1}{9}\cdot\frac{3\ln x + 1}{x^3} + C$$

$$I_1 = \lim_{t\to 0^+}\int_t^1 \frac{\ln x}{x^4}\,dx = -\frac{1}{9}\lim_{t\to 0^+}\left[\frac{3\ln x+1}{x^3}\right]_t^1 = -\frac{1}{9}\lim_{t\to 0^+}\left[1 - \frac{3\ln t + 1}{t^3}\right] = -\infty$$

So I_1 diverges and hence, I diverges. Divergent

57. $\int_{-\infty}^0 e^{-2x}\,dx = \lim_{t\to-\infty}\int_t^0 e^{-2x}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{2}e^{-2x}\right]_t^0 = \lim_{t\to-\infty}\left(-\frac{1}{2}+\frac{1}{2}e^{-2t}\right) = \infty$. Divergent

58. Note that $f(x) = 1/(2-3x)$ has an infinite discontinuity at $x = \frac{2}{3}$. Now

$$\int_0^{2/3}\frac{1}{2-3x}\,dx = \lim_{t\to(2/3)^-}\int_0^t \frac{1}{2-3x}\,dx = \lim_{t\to(2/3)^-}\left[-\tfrac{1}{3}\ln|2-3x|\right]_0^t$$
$$= -\tfrac{1}{3}\lim_{t\to(2/3)^-}\left[\ln|2-3t| - \ln 2\right] = \infty$$

Since $\displaystyle\int_0^{2/3}\frac{1}{2-3x}\,dx$ diverges, so does $\displaystyle\int_0^1 \frac{1}{2-3x}\,dx$.

59. Let $u = \ln x$. Then $du = dx/x$, so $\displaystyle\int \frac{dx}{x\sqrt{\ln x}} = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C = 2\sqrt{\ln x} + C$.

Thus, $\displaystyle\int_1^e \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to 1^+}\int_t^e \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to 1^+}\left[2\sqrt{\ln x}\right]_t^e = \lim_{t\to 1^+}\left(2\sqrt{\ln e} - 2\sqrt{\ln t}\right) = 2\cdot 1 - 2\cdot 0 = 2.$