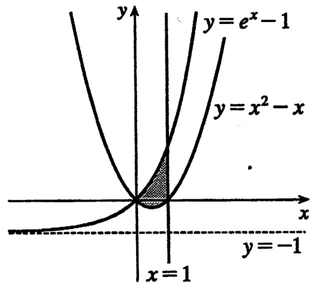
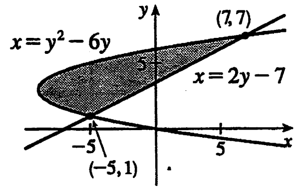
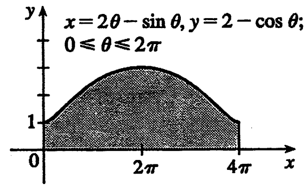
1. $A = \int_0^1 \left[(e^x - 1) - (x^2 - x)\right] dx$
$= \int_0^1 (e^x - 1 - x^2 + x)\, dx = \left[e^x - x - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^1$
$= \left(e - 1 - \frac{1}{3} + \frac{1}{2}\right) - (1 - 0 - 0 + 0) = e - \frac{11}{6}$



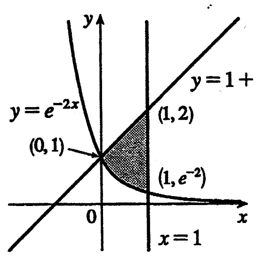
2. $A = \int_1^7 \left[(2y - 7) - (y^2 - 6y)\right] dy$
$= \int_1^7 (-y^2 + 8y - 7)\, dy$
$= \left[-\frac{1}{3}y^3 + 4y^2 - 7y\right]_1^7$
$= \left(-\frac{343}{3} + 196 - 49\right) - \left(-\frac{1}{3} + 4 - 7\right) = 36$



3. $x = 2\theta - \sin\theta \;\Rightarrow\; dx = (2 - \cos\theta)\, d\theta$
$A = \int_0^{2\pi} y\, dx = \int_0^{2\pi} \left[(2 - \cos\theta)(2 - \cos\theta)\right] d\theta$
$= \int_0^{2\pi} \left(4 - 4\cos\theta + \cos^2\theta\right) d\theta$
$= \int_0^{2\pi} \left(4 - 4\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$
$= \left[4\theta - 4\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$
$= (8\pi - 0 + \pi + 0) - (0) = 9\pi$



4. $V = \int_0^1 \pi\left[(1 + x)^2 - \left(e^{-2x}\right)^2\right] dx = \pi \int_0^1 \left(1 + 2x + x^2 - e^{-4x}\right) dx$
$= \pi\left[x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}e^{-4x}\right]_0^1 = \pi\left(1 + 1 + \frac{1}{3} + \frac{1}{4}e^{-4} - \frac{1}{4}\right)$
$= \pi\left(\dfrac{25}{12} + \dfrac{1}{4e^4}\right)$



5. (a) Using the Midpoint Rule on $[0, 1]$ with $f(x) = \tan(x^2)$ and $n = 4$, we estimate

$$A = \int_0^1 \tan(x^2)\, dx \approx \tfrac{1}{4}\left[\tan\left(\left(\tfrac{1}{8}\right)^2\right) + \tan\left(\left(\tfrac{3}{8}\right)^2\right) + \tan\left(\left(\tfrac{5}{8}\right)^2\right) + \tan\left(\left(\tfrac{7}{8}\right)^2\right)\right] \approx \tfrac{1}{4}(1.53) \approx 0.38$$

(b) Using the Midpoint Rule on $[0, 1]$ with $f(x) = \pi\tan^2(x^2)$ (for disks) and $n = 4$, we estimate

$$V = \int_0^1 f(x)\, dx \approx \tfrac{1}{4}\pi\left[\tan^2\left(\left(\tfrac{1}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{3}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{5}{8}\right)^2\right) + \tan^2\left(\left(\tfrac{7}{8}\right)^2\right)\right] \approx \tfrac{\pi}{4}(1.114) \approx 0.87$$

6. (a) $A = \int_0^1 (2x - x^2 - x^3)\, dx = \left[x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = 1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$

(b) A cross-section is a washer with inner radius x^3 and outer radius $2x - x^2$, so its area is $\pi(2x - x^2)^2 - \pi(x^3)^2$.

$$V = \int_0^1 A(x)\, dx = \int_0^1 \pi\left[(2x - x^2)^2 - (x^3)^2\right] dx = \int_0^1 \pi\left(4x^2 - 4x^3 + x^4 - x^6\right) dx$$
$$= \pi\left[\tfrac{4}{3}x^3 - x^4 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7\right]_0^1 = \pi\left(\tfrac{4}{3} - 1 + \tfrac{1}{5} - \tfrac{1}{7}\right) = \tfrac{41\pi}{105}$$

(c) Using the method of cylindrical shells,
$V = \int_0^1 2\pi x(2x - x^2 - x^3)\, dx = \int_0^1 2\pi(2x^2 - x^3 - x^4)\, dx = 2\pi\left[\frac{2}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1$
$= 2\pi\left(\frac{2}{3} - \frac{1}{4} - \frac{1}{5}\right) = \frac{13\pi}{30}$.

7. (a) A cross-section is a washer with inner radius x^2 and outer radius x.
$V = \int_0^1 \pi\left[(x)^2 - (x^2)^2\right] dx = \int_0^1 \pi(x^2 - x^4)\, dx = \pi\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1 = \pi\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2\pi}{15}$

(b) A cross-section is a washer with inner radius y and outer radius $\sqrt{y}$.
$V = \int_0^1 \pi\left[(\sqrt{y})^2 - y^2\right] dy = \int_0^1 \pi(y - y^2)\, dy = \pi\left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \pi\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\pi}{6}$

(c) A cross-section is a washer with inner radius $2 - x$ and outer radius $2 - x^2$.
$V = \int_0^1 \pi\left[(2 - x^2)^2 - (2 - x)^2\right] dx = \int_0^1 \pi(x^4 - 5x^2 + 4x)\, dx = \pi\left[\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2\right]_0^1$
$= \pi\left[\frac{1}{5} - \frac{5}{3} + 2\right] = \frac{8\pi}{15}$