5. $(3y^2+2y)\,y'=x\cos x \;\Rightarrow\; (3y^2+2y)\,dy=(x\cos x)\,dx \;\Rightarrow\; \int(3y^2+2y)\,dy=\int(x\cos x)\,dx \;\Rightarrow$ $y^3+y^2=\cos x+x\sin x+C$. For the last step, use integration by parts or Formula 83 in the Table of Integrals.

6. $\dfrac{dx}{dt}=1-t+x-tx=1(1-t)+x(1-t)=(1+x)(1-t) \;\Rightarrow\; \dfrac{dx}{1+x}=(1-t)\,dt \;\Rightarrow$
$\displaystyle\int\frac{dx}{1+x}=\int(1-t)\,dt \;\Rightarrow\; \ln|1+x|=t-\tfrac{1}{2}t^2+C \;\Rightarrow\; |1+x|=e^{t-t^2/2+C} \;\Rightarrow$
$1+x=\pm e^{t-t^2/2}\cdot e^C \;\Rightarrow\; x=-1+Ke^{t-t^2/2}$, where K is any nonzero constant.

7. $xyy'=\ln x \;\Rightarrow\; y\,dy=\dfrac{\ln x}{x}\,dx \;\Rightarrow\; \displaystyle\int y\,dy=\int\frac{\ln x}{x}\,dx$ (Make the substitution $u=\ln x$; then $du=dx/x$.) So $\int y\,dy=\int u\,du \;\Rightarrow\; \tfrac{1}{2}y^2=\tfrac{1}{2}u^2+C \;\Rightarrow \tfrac{1}{2}y^2=\tfrac{1}{2}(\ln x)^2+C$. $y(1)=2 \;\Rightarrow$ $\tfrac{1}{2}2^2=\tfrac{1}{2}(\ln 1)^2+C=C \;\Leftrightarrow\; C=2$. Therefore, $\tfrac{1}{2}y^2=\tfrac{1}{2}(\ln x)^2+2$, or $y=\sqrt{(\ln x)^2+4}$. The negative square root is inadmissible, since $y(1)>0$.

8. $1+x=2xyy' \;\Rightarrow\; y'=\dfrac{1+x}{2xy} \;\Leftrightarrow\; y\,dy=\dfrac{1+x}{2x}\,dx \;\Rightarrow\; \dfrac{y^2}{2}=\dfrac{\ln|x|}{2}+\dfrac{x}{2}+c_1$. But $x>0$, so $y^2=\ln x+x+c \;\Leftrightarrow\; y(x)=\pm\sqrt{c+x+\ln x}$. But $-2=y(1)$ so choose the negative square root and $-2=-\sqrt{c+1}$ so $c=3$. Thus, the solution is $y(x)=-\sqrt{3+x+\ln x}$.

9. The curves $kx^2+y^2=1$ form a family of ellipses for $k>0$, a family of hyperbolas for $k<0$, and two parallel lines $y=\pm1$ for $k=0$. Solving $kx^2+y^2=1$ for k gives $k=\dfrac{1-y^2}{x^2}$. Differentiating gives $2kx+2yy'=0 \;\Leftrightarrow$
$y'=-\dfrac{kx}{y}=-\left(1-y^2\right)\dfrac{x}{yx^2}=\dfrac{y^2-1}{xy}$. Thus, for $k\neq0$ the orthogonal trajectories must satisfy $y'=-\dfrac{xy}{y^2-1}$
$\Rightarrow\; \dfrac{y^2-1}{y}\,dy=-x\,dx \;\Rightarrow\; \dfrac{y^2}{2}-\ln|y|=\dfrac{-x^2}{2}+K \;\Rightarrow\; y^2-2\ln|y|+x^2=C$. For $k=0$, the orthogonal trajectories are given by $x=C_1$ for C_1 an arbitrary constant.

10. Differentiating both sides of $y=\dfrac{k}{1+x^2}$ gives $y'=-\dfrac{2kx}{(1+x^2)^2}=-2xy\dfrac{1+x^2}{(1+x^2)^2}=-\dfrac{2xy}{1+x^2}$. Thus, for $k\neq0$ the orthogonal trajectories must satisfy $y'=\dfrac{1+x^2}{2xy} \;\Rightarrow\; 2y\,dy=\left(\dfrac{1}{x}+x\right)dx \;\Rightarrow$
$y^2=\dfrac{x^2}{2}+\ln|x|+C$. For $k=0$, the orthogonal trajectories are given by $x=C_2$ for C_2 an arbitrary constant.

11. (a) $y(t)=y(0)e^{kt}=1000e^{kt} \;\Rightarrow\; y(2)=1000e^{2k}=9000 \;\Rightarrow\; e^{2k}=9 \;\Rightarrow\; 2k=\ln 9 \;\Rightarrow$
$k=\tfrac{1}{2}\ln 9=\ln 3 \;\Rightarrow\; y(t)=1000e^{(\ln 3)t}=1000\cdot 3^t$

(b) $y(3)=1000\cdot 3^3=27{,}000$

(c) $y'(t)=1000\cdot 3^t\cdot\ln 3$, so $y'(3)=27{,}000\ln 3\approx 29{,}663$ bacteria per hour

(d) $1000\cdot 3^t=2\cdot 1000 \;\Rightarrow\; 3^t=2 \;\Rightarrow\; t\ln 3=\ln 2 \;\Rightarrow\; t=(\ln 2)/\ln 3\approx 0.63$ h

12. (a) If $y(t)$ is the mass remaining after t years, then $y(t)=y(0)e^{kt}=18e^{kt}$. $y(25)=18e^{25k}=\tfrac{1}{2}\cdot 18 \;\Rightarrow$
$e^{25k}=\tfrac{1}{2} \;\Rightarrow\; 25k=-\ln 2 \;\Rightarrow\; k=-\tfrac{1}{25}\ln 2 \;\Rightarrow\; y(t)=18\,e^{-(\ln 2)t/25}=18\cdot 2^{-t/25}$.

(b) $18\cdot 2^{-t/25}=2 \;\Rightarrow\; 2^{-t/25}=\tfrac{1}{9} \;\Rightarrow\; -\tfrac{1}{25}t\ln 2=-\ln 9 \;\Rightarrow\; t=25\,\tfrac{\ln 9}{\ln 2}\approx 79$ years

13. (a) $C'(t)=-kC(t) \;\Rightarrow\; C(t)=C(0)e^{-kt}$ by Theorem 7.4.2. But $C(0)=C_0$, so $C(t)=C_0e^{-kt}$.

(b) $C(30)=\tfrac{1}{2}C_0$ since the concentration is reduced by half. Thus, $\tfrac{1}{2}C_0=C_0e^{-30k} \;\Rightarrow \ln\tfrac{1}{2}=-30k \;\Rightarrow$
$k=-\tfrac{1}{30}\ln\tfrac{1}{2}=\tfrac{1}{30}\ln 2$. Since 10% of the original concentration remains if 90% is eliminated, we want the value of t such that $C(t)=\tfrac{1}{10}C_0$. Therefore, $\tfrac{1}{10}C_0=C_0e^{-t(\ln 2)/30} \;\Rightarrow\; \ln 0.1=-t(\ln 2)/30 \;\Rightarrow$
$t=-\tfrac{30}{\ln 2}\ln 0.1\approx 100$ h.

16. Denote the amount of salt in the tank (in kg) by y. $y(0)=0$ since initially there is only water in the tank. The rate at which y increases is equal to the rate at which salt flows into the tank minus the rate at which it flows out. That rate is $\dfrac{dy}{dt}=0.1\dfrac{\text{kg}}{\text{L}}\times 10\dfrac{\text{L}}{\text{min}}-\dfrac{y}{100}\dfrac{\text{kg}}{\text{L}}\times 10\dfrac{\text{L}}{\text{min}}=1-\dfrac{y}{10}\dfrac{\text{kg}}{\text{min}} \;\Rightarrow\; \displaystyle\int\frac{dy}{10-y}=\int\frac{1}{10}\,dt \;\Rightarrow$
$-\ln|10-y|=\tfrac{1}{10}t+C \;\Rightarrow\; 10-y=Ae^{-t/10}$. $y(0)=0 \;\Rightarrow\; 10=A \;\Rightarrow\; y=10(1-e^{-t/10})$.
At $t=6$ minutes, $y=10\left(1-e^{-6/10}\right)\approx 4.512$ kg.