65. Let w be the width of the garden in feet. We use the perimeter to express the length of the garden in terms of width. Since the *perimeter* $= 2 \times$ *width* $+ 2 \times$ *length*, we have $200 = 2w + 2length \Leftrightarrow 2length = 200 - 2w \Leftrightarrow length = 100 - w$. Using the formula for area, we have $2400 = w(100 - w) = 100w - w^2 \Leftrightarrow w^2 - 100w + 2400 = 0 \Leftrightarrow (w - 40)(w - 60) = 0$. So $w - 40 = 0 \Leftrightarrow w = 40$, or $w - 60 = 0 \Leftrightarrow w = 60$. If $w = 40$, then $length = 100 - 40 = 60$. And if $w = 60$, then $length = 100 - 60 = 40$. So the length is 60 feet and the width is 40 feet.

72. Let h be the height of the flagpole, in feet. Then the length of each guy wire is $h + 5$. Since the distance between the points where the wires are fixed to the ground is equal to one guy wire, the triangle is equilateral, and the flagpole is the perpendicular bisector of the base. Thus from the Pythagorean Theorem, we get $\left[\frac{1}{2}(h+5)\right]^2 + h^2 = (h+5)^2 \Leftrightarrow$ $h^2 + 10h + 25 + 4h^2 = 4h^2 + 40h + 100 \Leftrightarrow h^2 - 30h - 75 = 0 \Rightarrow$ $h = \frac{-(-30)\pm\sqrt{(-30)^2-4(1)(-75)}}{2(1)} = \frac{30\pm\sqrt{900+300}}{2} = \frac{30 \pm \sqrt{1200}}{2} = \frac{30 \pm 20\sqrt{3}}{2}$. Since $h = \frac{30-20\sqrt{3}}{2} < 0$, we reject it. Thus the height is $h = \frac{30+20\sqrt{3}}{2} = 15 + 10\sqrt{3} \approx 32.32$ feet ≈ 32 feet 4 inches.

73. Using $h_0 = 288$, we solve $0 = -16t^2 + 288$, for $t \geq 0$. So $0 = -16t^2 + 288 \Leftrightarrow 16t^2 = 288 \Leftrightarrow t^2 = 18 \Rightarrow t = \pm\sqrt{18} = \pm 3\sqrt{2}$. Thus it takes $3\sqrt{2} \approx 4.24$ seconds for the ball the hit the ground.

78. Let y be the circumference of the circle, so $360 - y$ is the perimeter of the square. Use the circumference to find the radius, r, in terms of y: $y = 2\pi r \Rightarrow r = \dfrac{y}{2\pi}$. Thus the area of the circle is $\pi\left(\dfrac{y}{2\pi}\right)^2 = \dfrac{y^2}{4\pi}$. Now if the perimeter of the square is $360 - y$, the length of each side is

$\frac{1}{4}(360 - y)$, and the area of the square is $\left(\frac{360-y}{4}\right)^2$. Setting these areas equal, we obtain $\dfrac{y^2}{4\pi} = \left(\dfrac{360-y}{4}\right)^2 \Leftrightarrow \dfrac{y}{2\sqrt{\pi}} = \dfrac{360-y}{4} \Leftrightarrow 2y = 360\sqrt{\pi} - \sqrt{\pi}\,y \Leftrightarrow$ $(2 + \sqrt{\pi})y = 360\sqrt{\pi}$. Therefore, $y = \dfrac{360\sqrt{\pi}}{2 + \sqrt{\pi}} \approx 169.1$. Thus one wire is 169.1 in long and the other is 190.9 in long.

82. Let w be the uniform width of the lawn. With w cut off each end, the area of the factory is $(240 - 2w)(180 - 2w)$. Since the lawn and the factory are equal in size this area, is $\frac{1}{2} \cdot 240 \cdot 180$. So $21600 = 43200 - 480w - 360w + 4w^2 \Leftrightarrow 0 = 4w^2 - 840w + 21600$ $= 4(w^2 - 210w + 5400) = 4(w - 30)(w - 180) \Rightarrow w = 30$ or $w = 180$. Since 180 is too wide, the width of the lawn is 30 feet, and the factory is 120 by 180.