3.6 Exercises

1–2 ■
(a) Find y' by implicit differentiation.
(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.
(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

1. $xy + 2x + 3x^2 = 4$ **2.** $4x^2 + 9y^2 = 36$

3–12 ■ Find dy/dx by implicit differentiation.

3. $x^3 + x^2y + 4y^2 = 6$ **4.** $x^2 - 2xy + y^3 = c$

5. $x^2y + xy^2 = 3x$ **6.** $y^5 + x^2y^3 = 1 + ye^{x^2}$

7. $\sqrt{xy} = 1 + x^2y$ **8.** $\sqrt{1 + x^2y^2} = 2xy$

9. $4\cos x \sin y = 1$ **10.** $x\cos y + y\cos x = 1$

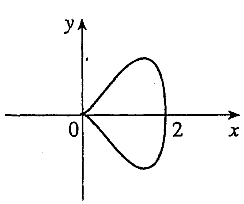
11. $\cos(x - y) = xe^x$ **12.** $\sin x + \cos y = \sin x \cos y$

13–18 ■ Find an equation of the tangent line to the curve at the given point.

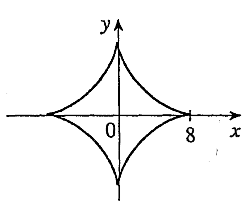
13. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1,\quad \left(-5, \tfrac{9}{4}\right)$ (hyperbola)

14. $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1,\quad \left(-1, 4\sqrt{2}\right)$ (ellipse)

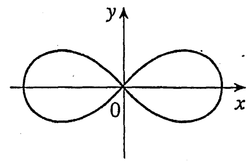
15. $y^2 = x^3(2 - x)$
$(1, 1)$
(piriform)



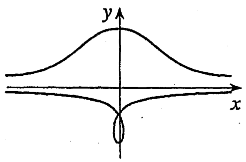
16. $x^{2/3} + y^{2/3} = 4$
$\left(-3\sqrt{3}, 1\right)$
(astroid)



17. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
$(3, 1)$
(lemniscate)



18. $x^2y^2 = (y + 1)^2(4 - y^2)$
$(0, -2)$
(conchoid of Nicomedes)



27–33 ■ Find the derivative of the function. Simplify where possible.

27. $y = \sin^{-1}(x^2)$ **28.** $y = (\sin^{-1}x)^2$

29. $y = 2\sqrt{x}\tan^{-1}\sqrt{x}$ **30.** $h(x) = \sqrt{1 - x^2}\arcsin x$

31. $H(x) = (1 + x^2)\arctan x$ **32.** $y = \tan^{-1}\left(x - \sqrt{1 + x^2}\right)$

33. $y = \arcsin(\tan\theta)$

34. The inverse cosine function $\cos^{-1} = \arccos$ is defined as the inverse of the restricted cosine function

$$f(x) = \cos x \qquad 0 \leqslant x \leqslant \pi$$

Therefore, $y = \cos^{-1}x$ means that $\cos y = x$ and $0 \leqslant y \leqslant \pi$. Show that

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$$

49. Find all points on the curve $x^2y^2 + xy = 2$ where the slop of the tangent line is -1.

50. Find the equations of both the tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$.

51. (a) Suppose f is a one-to-one differentiable function and i inverse function f^{-1} is also differentiable. Use implicit differentiation to show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

provided that the denominator is not 0.
(b) If $f(4) = 5$ and $f'(4) = \frac{2}{3}$, find $(f^{-1})'(5)$.

52. (a) Show that $f(x) = 2x + \cos x$ is one-to-one.
(b) What is the value of $f^{-1}(1)$?
(c) Use the formula from Exercise 51(a) to find $(f^{-1})'(1)$.

53. The **Bessel function** of order 0, $y = J(x)$, satisfies the differential equation $xy'' + y' + xy = 0$ for all values of x an its value at 0 is $J(0) = 1$.
(a) Find $J'(0)$.
(b) Use implicit differentiation to find $J''(0)$.

54. The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \leqslant 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?