

49.  $x^2y^2 + xy = 2 \Rightarrow x^2 \cdot 2yy' + y^2 \cdot 2x + x \cdot y' + y \cdot 1 = 0 \Leftrightarrow y'(2x^2y + x) = -2xy^2 - y \Leftrightarrow$   
 $y' = -\frac{2xy^2 + y}{2x^2y + x}$ . So  $-\frac{2xy^2 + y}{2x^2y + x} = -1 \Leftrightarrow 2xy^2 + y = 2x^2y + x \Leftrightarrow y(2xy + 1) = x(2xy + 1) \Leftrightarrow$   
 $y(2xy + 1) - x(2xy + 1) = 0 \Leftrightarrow (2xy + 1)(y - x) = 0 \Leftrightarrow xy = -\frac{1}{2}$  or  $y = x$ . But  $xy = -\frac{1}{2} \Rightarrow$   
 $x^2y^2 + xy = \frac{1}{4} - \frac{1}{2} \neq 2$ , so we must have  $x = y$ . Then  $x^2y^2 + xy = 2 \Rightarrow x^4 + x^2 = 2 \Leftrightarrow$   
 $x^4 + x^2 - 2 = 0 \Leftrightarrow (x^2 + 2)(x^2 - 1) = 0$ . So  $x^2 = -2$ , which is impossible, or  $x^2 = 1 \Leftrightarrow x = \pm 1$ .  
 Since  $x = y$ , the points on the curve where the tangent line has a slope of  $-1$  are  $(-1, -1)$  and  $(1, 1)$ .

54.  $x^2 + 4y^2 = 5 \Rightarrow 2x + 4(2yy') = 0 \Rightarrow y' = -\frac{x}{4y}$ . Now let  $h$  be the height of the lamp, and let  $(a, b)$  be the point of tangency of the line passing through the points  $(3, h)$  and  $(-5, 0)$ . This line has slope  $(h - 0) / [3 - (-5)] = \frac{1}{8}h$ . But the slope of the tangent line through the point  $(a, b)$  can be expressed as  $y' = -\frac{a}{4b}$ , or as  $\frac{b - 0}{a - (-5)} = \frac{b}{a + 5}$  [since the line passes through  $(-5, 0)$  and  $(a, b)$ ], so  $-\frac{a}{4b} = \frac{b}{a + 5} \Leftrightarrow$   
 $4b^2 = -a^2 - 5a \Leftrightarrow a^2 + 4b^2 = -5a$ . But  $a^2 + 4b^2 = 5$  [since  $(a, b)$  is on the ellipse], so  $5 = -5a \Leftrightarrow$   
 $a = -1$ . Then  $4b^2 = -a^2 - 5a = -1 - 5(-1) = 4 \Rightarrow b = 1$ , since the point is on the top half of the ellipse.  
 So  $\frac{h}{8} = \frac{b}{a + 5} = \frac{1}{-1 + 5} = \frac{1}{4} \Rightarrow h = 2$ . So the lamp is located 2 units above the  $x$ -axis.