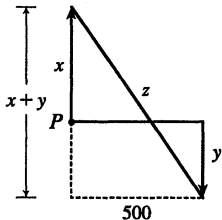


11.



We are given that $\frac{dx}{dt} = 4$ ft/s and $\frac{dy}{dt} = 5$ ft/s. $z^2 = (x + y)^2 + 500^2 \Rightarrow$

$$2z \frac{dz}{dt} = 2(x + y) \left(\frac{dx}{dt} + \frac{dy}{dt} \right). \text{ 15 minutes after the woman starts, we have}$$

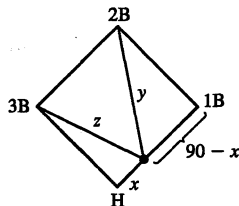
$$x = (4 \text{ ft/s})(20 \text{ min})(60 \text{ s/min}) = 4800 \text{ ft and } y = 5 \cdot 15 \cdot 60 = 4500 \Rightarrow$$

$$z = \sqrt{(4800 + 4500)^2 + 500^2} = \sqrt{86,740,000}, \text{ so}$$

$$\frac{dz}{dt} = \frac{x + y}{z} \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = \frac{4800 + 4500}{\sqrt{86,740,000}} (5 + 4) = \frac{837}{\sqrt{8674}} \approx 8.99 \text{ ft/s.}$$

12. We are given that $\frac{dx}{dt} = 24$ ft/s.

(a)



$$y^2 = (90 - x)^2 + 90^2 \Rightarrow 2y \frac{dy}{dt} = 2(90 - x) \left(-\frac{dx}{dt} \right).$$

When $x = 45$, $y = \sqrt{45^2 + 90^2} = 45\sqrt{5}$, so

$$\frac{dy}{dt} = \frac{90 - x}{y} \left(-\frac{dx}{dt} \right) = \frac{45}{45\sqrt{5}} (-24) = -\frac{24}{\sqrt{5}},$$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$$z^2 = x^2 + 90^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt}. \text{ When } x = 45, z = 45\sqrt{5}, \text{ so } \frac{dz}{dt} = \frac{45}{45\sqrt{5}} (24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s.}$$

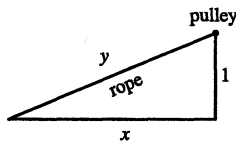
13. $A = \frac{1}{2}bh$, where b is the base and h is the altitude. We are given that $\frac{dh}{dt} = 1$ cm/min and $\frac{dA}{dt} = 2$ cm²/min.

Using the Product Rule, we have $\frac{dA}{dt} = \frac{1}{2} \left(b \frac{dh}{dt} + h \frac{db}{dt} \right)$. When $h = 10$ and $A = 100$, we have

$$100 = \frac{1}{2}b(10) \Rightarrow \frac{1}{2}b = 10 \Rightarrow b = 20, \text{ so } 2 = \frac{1}{2} \left(20 \cdot 1 + 10 \frac{db}{dt} \right) \Rightarrow 4 = 20 + 10 \frac{db}{dt} \Rightarrow$$

$$\frac{db}{dt} = \frac{4 - 20}{10} = -1.6 \text{ cm/min.}$$

14.

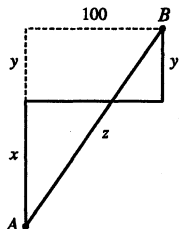


Given $\frac{dy}{dt} = -1$ m/s, find $\frac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \Rightarrow$

$$2y \frac{dy}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt} = -\frac{y}{x}. \text{ When } x = 8, y = \sqrt{65}, \text{ so}$$

$$\frac{dx}{dt} = -\frac{\sqrt{65}}{8}. \text{ Thus, the boat approaches the dock at } \frac{\sqrt{65}}{8} \approx 1.01 \text{ m/s.}$$

15.



We are given that $\frac{dx}{dt} = 35$ km/h and $\frac{dy}{dt} = 25$ km/h. $z^2 = (x + y)^2 + 100^2$

$$\Rightarrow 2z \frac{dz}{dt} = 2(x + y) \left(\frac{dx}{dt} + \frac{dy}{dt} \right). \text{ At 4:00 P.M., } x = 4(35) = 140 \text{ and}$$

$$y = 4(25) = 100 \Rightarrow z = \sqrt{(140 + 100)^2 + 100^2} = \sqrt{67,600} = 260, \text{ so}$$

$$\frac{dz}{dt} = \frac{x + y}{z} \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = \frac{140 + 100}{260} (35 + 25) = \frac{720}{13} \approx 55.4 \text{ km/h.}$$