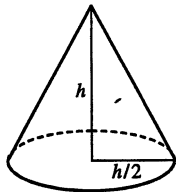


21.

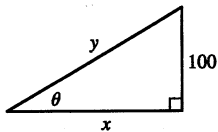


$$\text{We are given that } \frac{dV}{dt} = 30 \text{ ft}^3/\text{min}. V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$$

$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \Rightarrow 30 = \frac{\pi h^2}{4} \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{120}{\pi h^2}. \text{ When}$$

$$h = 10 \text{ ft}, \frac{dh}{dt} = \frac{120}{10^2\pi} = \frac{6}{5\pi} \approx 0.38 \text{ ft/min.}$$

22.

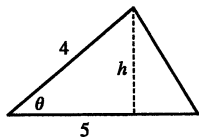


$$\text{We are given } dx/dt = 8 \text{ ft/s. } \cot \theta = \frac{x}{100} \Rightarrow x = 100 \cot \theta \Rightarrow$$

$$\frac{dx}{dt} = -100 \csc^2 \theta \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{\sin^2 \theta}{100} \cdot 8. \text{ When } y = 200,$$

$$\sin \theta = \frac{100}{200} = \frac{1}{2} \Rightarrow \frac{d\theta}{dt} = -\frac{(1/2)^2}{100} \cdot 8 = -\frac{1}{50} \text{ rad/s. The angle is decreasing at a rate of } \frac{1}{50} \text{ rad/s.}$$

23.



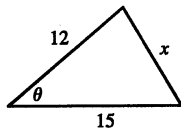
$$A = \frac{1}{2}bh, \text{ but } b = 5 \text{ m and } \sin \theta = \frac{h}{4} \Rightarrow h = 4 \sin \theta, \text{ so}$$

$$A = \frac{1}{2}(5)(4 \sin \theta) = 10 \sin \theta. \text{ We are given } \frac{d\theta}{dt} = 0.06 \text{ rad/s, so}$$

$$\frac{dA}{dt} = \frac{dA}{d\theta} \frac{d\theta}{dt} = 10 \cos \theta \frac{d\theta}{dt} = 0.6 \cos \theta. \text{ When } \theta = \frac{\pi}{3},$$

$$\frac{dA}{dt} = 0.6(\cos \frac{\pi}{3}) = (0.6)(\frac{1}{2}) = 0.3 \text{ m}^2/\text{s}.$$

24.



$$\text{We are given } d\theta/dt = 2^\circ/\text{min} = \frac{\pi}{90} \text{ rad/min. By the Law of Cosines,}$$

$$x^2 = 12^2 + 15^2 - 2(12)(15) \cos \theta = 369 - 360 \cos \theta \Rightarrow$$

$$2x \frac{dx}{dt} = 360 \sin \theta \frac{d\theta}{dt} \Rightarrow \frac{dx}{dt} = \frac{180 \sin \theta}{x} \frac{d\theta}{dt}. \text{ When } \theta = 60^\circ,$$

$$x = \sqrt{369 - 360 \cos 60^\circ} = \sqrt{189} = 3\sqrt{21}, \text{ so}$$

$$\frac{dx}{dt} = \frac{180 \sin 60^\circ}{3\sqrt{21}} \frac{\pi}{90} = \frac{\pi \sqrt{3}}{3\sqrt{21}} = \frac{\sqrt{7}\pi}{21} \approx 0.396 \text{ m/min.}$$