

5. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.
6. Find the dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible.
7. Consider the following problem: A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?
- (a) Draw several diagrams illustrating the situation, some with shallow, wide pens and some with deep, narrow pens. Find the total areas of these configurations. Does it appear that there is a maximum area? If so, estimate it.
- (b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.
- (c) Write an expression for the total area.
- (d) Use the given information to write an equation that relates the variables.
- (e) Use part (d) to write the total area as a function of one variable.
- (f) Finish solving the problem and compare the answer with your estimate in part (a).
8. Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.
- (a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes. Does it appear that there is a maximum volume? If so, estimate it.
- (b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.
- (c) Write an expression for the volume.
- (d) Use the given information to write an equation that relates the variables.
- (e) Use part (d) to write the volume as a function of one variable.
- (f) Finish solving the problem and compare the answer with your estimate in part (a).

9. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
10. A box with a square base and open top must have a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used.
11. (a) Show that of all the rectangles with a given area, the one with smallest perimeter is a square.
- (b) Show that of all the rectangles with a given perimeter, the one with greatest area is a square.
12. A rectangular storage container with an open top is to have a volume of 10 m^3 . The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.
13. Find the point on the line $y = 4x + 7$ that is closest to the origin.
14. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.
15. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.
16. Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 8 - x^2$.
17. A right circular cylinder is inscribed in a sphere of radius r . Find the largest possible surface area of such a cylinder.
18. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.
19. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See Exercise 48 on page 24.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

20. A right circular cylinder is inscribed in a cone with height h and base radius r . Find the largest possible volume of such a cylinder.
21. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?
22. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?
25. In a beehive, each cell is a regular hexagonal prism, open at one end with a trihedral angle at the other end. It is believed that bees form their cells in such a way as to minimize the surface area for a given volume, thus using the least amount of wax in cell construction. Examination of these cells has shown that the measure of the apex angle θ is amazingly consistent. Based on the geometry of the cell, it can be shown that the surface area S is given by

$$S = 6sh - \frac{3}{2}s^2 \cot \theta + (3s^2\sqrt{3}/2) \csc \theta$$

where s , the length of the sides of the hexagon, and h , the height, are constants.

(a) Calculate $dS/d\theta$.

(b) What angle should the bees prefer?

(c) Determine the minimum surface area of the cell (in terms of s and h).

Note: Actual measurements of the angle θ in beehives have been made, and the measures of these angles seldom differ from the calculated value by more than 2° .

