

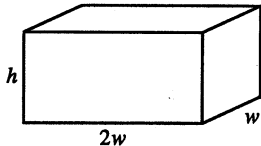
9. Let b be the length of the base of the box and h the height. The surface area is $1200 = b^2 + 4hb \Rightarrow h = (1200 - b^2)/(4b)$. The volume is $V = b^2h = b^2(1200 - b^2)/4b = 300b - b^3/4 \Rightarrow V'(b) = 300 - \frac{3}{4}b^2$. $V'(b) = 0 \Rightarrow 300 = \frac{3}{4}b^2 \Rightarrow b^2 = 400 \Rightarrow b = \sqrt{400} = 20$. Since $V'(b) > 0$ for $0 < b < 20$ and $V'(b) < 0$ for $b > 20$, there is an absolute maximum when $b = 20$ by the First Derivative Test for Absolute Extreme Values (see page 310). If $b = 20$, then $h = (1200 - 20^2)/(4 \cdot 20) = 10$, so the largest possible volume is $b^2h = (20)^2(10) = 4000 \text{ cm}^3$.

10. Let b be the length of the base of the box and h the height. The volume is $32,000 = b^2h \Rightarrow h = 32,000/b^2$. The surface area of the open box is $b^2 + 4hb = b^2 + 4(32,000/b^2)b = b^2 + 4(32,000)/b$. So $V'(b) = 2b - 4(32,000)/b^2 = 2(b^3 - 64,000)/b^2 = 0 \Leftrightarrow b = \sqrt[3]{64,000} = 40$. This gives an absolute minimum since $V'(b) < 0$ if $0 < b < 40$ and $V'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

11. (a) Let the rectangle have sides x and y and area A , so $A = xy$ or $y = A/x$. The problem is to minimize the perimeter $= 2x + 2y = 2x + 2A/x = P(x)$. Now $P'(x) = 2 - 2A/x^2 = 2(x^2 - A)/x^2$. So the critical number is $x = \sqrt{A}$. Since $P'(x) < 0$ for $0 < x < \sqrt{A}$ and $P'(x) > 0$ for $x > \sqrt{A}$, there is an absolute minimum at $x = \sqrt{A}$. The sides of the rectangle are \sqrt{A} and $A/\sqrt{A} = \sqrt{A}$, so the rectangle is a square.

(b) Let p be the perimeter and x and y the lengths of the sides, so $p = 2x + 2y \Rightarrow 2y = p - 2x \Rightarrow y = \frac{1}{2}p - x$. The area is $A(x) = x(\frac{1}{2}p - x) = \frac{1}{2}px - x^2$. Now $A'(x) = 0 \Rightarrow \frac{1}{2}p - 2x = 0 \Rightarrow 2x = \frac{1}{2}p \Rightarrow x = \frac{1}{4}p$. Since $A''(x) = -2 < 0$, there is an absolute maximum for A when $x = \frac{1}{4}p$ by the Second Derivative Test. The sides of the rectangle are $\frac{1}{4}p$ and $\frac{1}{2}p - \frac{1}{4}p = \frac{1}{4}p$, so the rectangle is a square.

12.



$V = lwh \Rightarrow 10 = (2w)(w)h = 2w^2h$, so $h = 5/w^2$. The cost is

$$10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh, \text{ so}$$

$$C(w) = 20w^2 + 36w(5/w^2) = 20w^2 + 180/w.$$

$$C'(w) = 40w - 180/w^2 = 40(w^3 - \frac{9}{2})/w^2 \Rightarrow w = \sqrt[3]{\frac{9}{2}} \text{ is the}$$

critical number. There is an absolute minimum for C when $w = \sqrt[3]{\frac{9}{2}}$ since $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{9}{2}}$ and

$$C'(w) > 0 \text{ for } w > \sqrt[3]{\frac{9}{2}}. \quad C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{9/2}} \approx \$163.54.$$