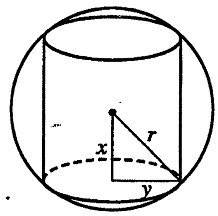
17.



The cylinder has surface area
2(area of the base) + (lateral surface area)
$= 2\pi(\text{radius})^2 + 2\pi(\text{radius})(\text{height}) = 2\pi y^2 + 2\pi y(2x)$.
Now $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow y = \sqrt{r^2 - x^2}$, so the surface area is

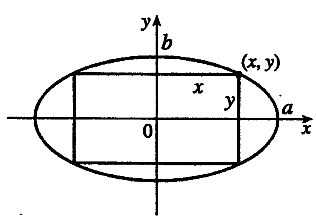
$$S(x) = 2\pi(r^2 - x^2) + 4\pi x\sqrt{r^2 - x^2}, \quad 0 \le x \le r$$
$$= 2\pi r^2 - 2\pi x^2 + 4\pi\left(x\sqrt{r^2 - x^2}\right)$$

Thus, $S'(x) = 0 - 4\pi x + 4\pi\left[x \cdot \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x) + (r^2 - x^2)^{1/2} \cdot 1\right]$

$$= 4\pi\left[-x - \frac{x^2}{\sqrt{r^2 - x^2}} + \sqrt{r^2 - x^2}\right] = 4\pi \cdot \frac{-x\sqrt{r^2 - x^2} - x^2 + r^2 - x^2}{\sqrt{r^2 - x^2}}$$

$S'(x) = 0 \Rightarrow x\sqrt{r^2 - x^2} = r^2 - 2x^2 \quad (\star) \Rightarrow \left(x\sqrt{r^2 - x^2}\right)^2 = (r^2 - 2x^2)^2 \Rightarrow$
$x^2(r^2 - x^2) = r^4 - 4r^2x^2 + 4x^4 \Rightarrow r^2x^2 - x^4 = r^4 - 4rx^2 + 4x^4 \Rightarrow 5x^4 - 5r^2x^2 + r^4 = 0$. This is a quadratic equation in x^2. By the quadratic formula, $x^2 = \frac{5 \pm \sqrt{5}}{10}r^2$, but we reject the root with the + sign since it doesn't satisfy $(\star)$. [The right side is negative and the left side is positive.] So $x = \sqrt{\frac{5 - \sqrt{5}}{10}}\, r$. Since $S(0) = S(r) = 0$, the maximum surface area occurs at the critical number and
$x^2 = \frac{5 - \sqrt{5}}{10}r^2 \Rightarrow y^2 = r^2 - \frac{5 - \sqrt{5}}{10}r^2 = \frac{5 + \sqrt{5}}{10}r^2 \Rightarrow$ the surface area is
$2\pi\left(\frac{5 + \sqrt{5}}{10}\right)r^2 + 4\pi\sqrt{\frac{5 - \sqrt{5}}{10}}\sqrt{\frac{5 + \sqrt{5}}{10}}r^2 = \pi r^2\left[2 \cdot \frac{5 + \sqrt{5}}{10} + 4\frac{\sqrt{(5 - \sqrt{5})(5 + \sqrt{5})}}{10}\right] = \pi r^2\left[\frac{5 + \sqrt{5}}{5} + \frac{2\sqrt{20}}{5}\right] =$
$\pi r^2\left[\frac{5 + \sqrt{5} + 2 \cdot 2\sqrt{5}}{5}\right] = \pi r^2\left[\frac{5 + 5\sqrt{5}}{5}\right] = \pi r^2(1 + \sqrt{5})$.

18.

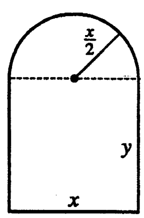


The area of the rectangle is $(2x)(2y) = 4xy$. Now $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ gives $y = \frac{b}{a}\sqrt{a^2 - x^2}$, so we maximize $A(x) = 4\frac{b}{a}x\sqrt{a^2 - x^2}$.

$$A'(x) = \frac{4b}{a}\left[x \cdot \frac{1}{2}(a^2 - x^2)^{-1/2}(-2x) + (a^2 - x^2)^{1/2} \cdot 1\right]$$
$$= \frac{4b}{a}(a^2 - x^2)^{-1/2}\left[-x^2 + a^2 - x^2\right] = \frac{4b}{a\sqrt{a^2 - x^2}}\left[a^2 - 2x^2\right]$$

So the critical number is $x = \frac{1}{\sqrt{2}}a$, and this clearly gives a maximum. Then $y = \frac{1}{\sqrt{2}}b$, so the maximum area is $4\left(\frac{1}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}b\right) = 2ab$.

19.



Perimeter $= 30 \Rightarrow 2y + x + \pi\left(\frac{x}{2}\right) = 30 \Rightarrow$
$y = \frac{1}{2}\left(30 - x - \frac{\pi x}{2}\right) = 15 - \frac{x}{2} - \frac{\pi x}{4}$. The area is the area of the rectangle plus the area of the semicircle, or $xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$, so
$A(x) = x\left(15 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{8}\pi x^2 = 15x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$.

$A'(x) = 15 - \left(1 + \frac{\pi}{4}\right)x = 0 \Rightarrow x = \frac{15}{1 + \pi/4} = \frac{60}{4 + \pi}$. $A''(x) = -\left(1 + \frac{\pi}{4}\right) < 0$, so this gives a maximum. The dimensions are $x = \frac{60}{4 + \pi}$ ft and $y = 15 - \frac{30}{4 + \pi} - \frac{15\pi}{4 + \pi} = \frac{60 + 15\pi - 30 - 15\pi}{4 + \pi} = \frac{30}{4 + \pi}$ ft, so the height of the rectangle is half the base.