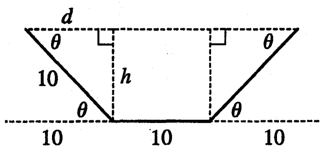
38.

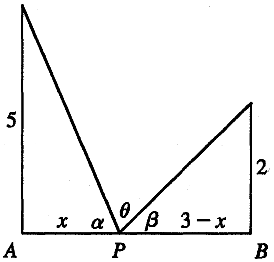


We maximize the cross-sectional area

$$A(\theta) = 10h + 2\left(\tfrac{1}{2}dh\right) = 10h + dh = 10(10\sin\theta) + (10\cos\theta)(10\sin\theta)$$
$$= 100(\sin\theta + \sin\theta\,\cos\theta),\quad 0 \le \theta \le \tfrac{\pi}{2}$$
$$A'(\theta) = 100\left(\cos\theta + \cos^2\theta - \sin^2\theta\right) = 100\left(\cos\theta + 2\cos^2\theta - 1\right) = 100(2\cos\theta - 1)(\cos\theta + 1)$$
$$= 0 \text{ when } \cos\theta = \tfrac{1}{2} \quad\Leftrightarrow\quad \theta = \tfrac{\pi}{3}. \quad (\cos\theta \ne -1 \text{ since } 0 \le \theta \le \tfrac{\pi}{2}.)$$

Now $A(0) = 0$, $A\left(\frac{\pi}{2}\right) = 100$ and $A\left(\frac{\pi}{3}\right) = 75\sqrt{3} \approx 129.9$, so the maximum occurs when $\theta = \frac{\pi}{3}$.

39.



From the figure, $\tan\alpha = \dfrac{5}{x}$ and $\tan\beta = \dfrac{2}{3-x}$. Since $\alpha + \beta + \theta = 180^\circ = \pi$, $\theta = \pi - \tan^{-1}\left(\dfrac{5}{x}\right) - \tan^{-1}\left(\dfrac{2}{3-x}\right) \Rightarrow$

$$\frac{d\theta}{dx} = -\frac{1}{1+\left(\frac{5}{x}\right)^2}\left(-\frac{5}{x^2}\right) - \frac{1}{1+\left(\frac{2}{3-x}\right)^2}\left[\frac{2}{(3-x)^2}\right]$$
$$= \frac{x^2}{x^2+25}\cdot\frac{5}{x^2} - \frac{(3-x)^2}{(3-x)^2+4}\cdot\frac{2}{(3-x)^2}.$$ Now

$$\frac{d\theta}{dx} = 0 \quad\Rightarrow\quad \frac{5}{x^2+25} = \frac{2}{x^2-6x+13} \quad\Rightarrow\quad 2x^2 + 50 = 5x^2 - 30x + 65 \quad\Rightarrow$$