4.9 Exercises

1–12 ■ Find the most general antiderivative of the function. (Check your answer by differentiation.)

1. $f(x) = 6x^2 - 8x + 3$

2. $f(x) = 1 - x^3 + 12x^5$

3. $f(x) = 5x^{1/4} - 7x^{3/4}$

4. $f(x) = 2x + 3x^{1.7}$

5. $f(x) = \dfrac{10}{x^9}$

6. $f(x) = \sqrt[3]{x^2} - \sqrt{x^3}$

7. $g(t) = \dfrac{t^3 + 2t^2}{\sqrt{t}}$

8. $f(x) = \dfrac{3}{x^2} - \dfrac{5}{x^4}$

9. $f(t) = 3\cos t - 4\sin t$

10. $f(x) = 3e^x + 7\sec^2 x$

11. $f(x) = 2x + 5(1 - x^2)^{-1/2}$

12. $f(x) = \dfrac{x^2 + x + 1}{x}$

13–14 ■ Find the antiderivative F of f that satisfies the given condition. Check your answer by comparing the graphs of f and F.

13. $f(x) = 5x^4 - 2x^5, \quad F(0) = 4$

14. $f(x) = 4 - 3(1 + x^2)^{-1}, \quad F(1) = 0$

15–24 ■ Find f.

15. $f''(x) = 6x + 12x^2$

16. $f''(x) = 2 + x^3 + x^6$

17. $f''(x) = 1 + x^{4/5}$

18. $f''(x) = \cos x$

19. $f'(x) = 3\cos x + 5\sin x, \quad f(0) = 4$

20. $f'(x) = 4/\sqrt{1 - x^2}, \quad f(\frac{1}{2}) = 1$

21. $f''(x) = x, \quad f(0) = -3, \quad f'(0) = 2$

22. $f''(x) = x + \sqrt{x}, \quad f(1) = 1, \quad f'(1) = 2$

23. $f''(x) = x^{-2}, \quad x > 0, \quad f(1) = 0, \quad f(2) = 0$

24. $f''(x) = 3e^x + 5\sin x, \quad f(0) = 1, \quad f'(0) = 2$

35. A particle moves along a straight line with velocity function $v(t) = \sin t - \cos t$ and its initial displacement is $s(0) = 0$ m. Find its position function $s(t)$.

36. A particle moves with acceleration function $a(t) = 5 + 4t - 2t^2$. Its initial velocity is $v(0) = 3$ m/s and its initial displacement is $s(0) = 10$ m. Find its position after t seconds.

37. A stone is dropped from the upper observation deck (the Space Deck) of the CN Tower, 450 m above the ground.
(a) Find the distance of the stone above ground level at time t.
(b) How long does it take the stone to reach the ground?
(c) With what velocity does it strike the ground?
(d) If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?

38. Show that for motion in a straight line with constant acceleration a, initial velocity v_0, and initial displacement s_0, the displacement after time t is

$$s = \tfrac{1}{2}at^2 + v_0 t + s_0$$

39. An object is projected upward with initial velocity v_0 meters per second from a point s_0 meters above the ground. Show that

$$[v(t)]^2 = v_0^2 - 19.6[s(t) - s_0]$$

40. Two balls are thrown upward from the edge of the cliff in Example 7. The first is thrown with a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?

41. A company estimates that the marginal cost (in dollars per item) of producing x items is $1.92 - 0.002x$. If the cost of producing one item is \$562, find the cost of producing 100 items.

42. The linear density of a rod of length 1 m is given by $\rho(x) = 1/\sqrt{x}$, in grams per centimeter, where x is measured in centimeters from one end of the rod. Find the mass of the rod.

43. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?

44. A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 40 ft/s^2. What is the distance covered before the car comes to a stop?

45. What constant acceleration is required to increase the speed of a car from 30 mi/h to 50 mi/h in 5 s?

46. A car braked with a constant deceleration of 40 ft/s^2, producing skid marks measuring 160 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

47. To prove Theorem 1, let F and G be any two antiderivatives of f on I and let $H = G - F$.
(a) If x_1 and x_2 are any two numbers in I with $x_1 < x_2$, apply the Mean Value Theorem on the interval $[x_1, x_2]$ to show that $H(x_1) = H(x_2)$. Why does this show that H is a constant function?
(b) Deduce Theorem 1 from the result of part (a).

48. Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of 10 m/s and its downward acceleration is

$$a = \begin{cases} 9 - 0.9t & \text{if } 0 \le t \le 10 \\ 0 & \text{if } t > 10 \end{cases}$$

If the raindrop is initially 500 m above the ground, how long does it take to fall?

49. A high-speed "bullet" train accelerates and decelerates at the rate of 4 ft/s^2. Its maximum cruising speed is 90 mi/h.
(a) What is the maximum distance the train can travel if it accelerates from rest until it reaches its cruising speed and then runs at that speed for 15 minutes?
(b) Suppose that the train starts from rest and must come to a complete stop in 15 minutes. What is the maximum distance it can travel under these conditions?
(c) Find the minimum time that the train takes to travel between two consecutive stations that are 45 miles apart.
(d) The trip from one station to the next takes 37.5 minutes. How far apart are the stations?