4.9 Antiderivatives

1. $f(x) = 6x^2 - 8x + 3 \quad \Rightarrow \quad F(x) = 6\dfrac{x^{2+1}}{2+1} - 8\dfrac{x^{1+1}}{1+1} + 3x + C = 2x^3 - 4x^2 + 3x + C$

Check: $F'(x) = 2 \cdot 3x^2 - 4 \cdot 2x + 3 + 0 = 6x^2 - 8x + 3 = f(x)$

2. $f(x) = 1 - x^3 + 12x^5 \quad \Rightarrow \quad F(x) = x - \dfrac{x^{3+1}}{3+1} + 12\dfrac{x^{5+1}}{5+1} + C = x - \frac{1}{4}x^4 + 2x^6 + C$

3. $f(x) = 5x^{1/4} - 7x^{3/4} \quad \Rightarrow \quad F(x) = 5\dfrac{x^{1/4+1}}{\frac{1}{4}+1} - 7\dfrac{x^{3/4+1}}{\frac{3}{4}+1} + C = 5\dfrac{x^{5/4}}{5/4} - 7\dfrac{x^{7/4}}{7/4} + C = 4x^{5/4} - 4x^{7/4} + C$

Check: $F'(x) = 4 \cdot \frac{5}{4}x^{1/4} - 4 \cdot \frac{7}{4}x^{3/4} = 5x^{1/4} - 7x^{3/4} = f(x)$

4. $f(x) = 2x + 3x^{1.7} \quad \Rightarrow \quad F(x) = x^2 + \frac{3}{2.7}x^{2.7} + C = x^2 + \frac{10}{9}x^{2.7} + C$

5. $f(x) = \dfrac{10}{x^9} = 10x^{-9}$ has domain $(-\infty, 0) \cup (0, \infty)$, so $F(x) = \begin{cases} \dfrac{10x^{-8}}{-8} + C_1 = -\dfrac{5}{4x^8} + C_1 & \text{if } x < 0 \\ -\dfrac{5}{4x^8} + C_2 & \text{if } x > 0 \end{cases}$

See Example 1(b) for a similar exercise.

6. $f(x) = \sqrt[3]{x^2} - \sqrt{x^3} = x^{2/3} - x^{3/2} \quad \Rightarrow \quad F(x) = \frac{1}{5/3}x^{5/3} - \frac{1}{5/2}x^{5/2} + C = \frac{3}{5}x^{5/3} - \frac{2}{5}x^{5/2} + C$

7. $g(t) = \dfrac{t^3 + 2t^2}{\sqrt{t}} = t^{5/2} + 2t^{3/2} \quad \Rightarrow \quad G(t) = \dfrac{t^{7/2}}{7/2} + \dfrac{2t^{5/2}}{5/2} + C = \frac{2}{7}t^{7/2} + \frac{4}{5}t^{5/2} + C$

Note that g has domain $(0, \infty)$.

8. $f(x) = \dfrac{3}{x^2} - \dfrac{5}{x^4} = 3x^{-2} - 5x^{-4}$ has domain $(-\infty, 0) \cup (0, \infty)$, so

$$F(x) = \begin{cases} \dfrac{3x^{-1}}{-1} - \dfrac{5x^{-3}}{-3} + C_1 = -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_1 & \text{if } x < 0 \\ -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_2 & \text{if } x > 0 \end{cases}$$

9. $f(t) = 3\cos t - 4\sin t \quad \Rightarrow \quad F(t) = 3(\sin t) - 4(-\cos t) + C = 3\sin t + 4\cos t + C$

10. $f(x) = 3e^x + 7\sec^2 x \quad \Rightarrow \quad F(x) = 3e^x + 7\tan x + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

11. $f(x) = 2x + 5\left(1 - x^2\right)^{-1/2} = 2x + \dfrac{5}{\sqrt{1 - x^2}} \quad \Rightarrow \quad F(x) = x^2 + 5\sin^{-1} x + C$

12. $f(x) = \dfrac{x^2 + x + 1}{x} = x + 1 + \dfrac{1}{x} \quad \Rightarrow \quad F(x) = \begin{cases} \frac{1}{2}x^2 + x + \ln|x| + C_1 & \text{if } x < 0 \\ \frac{1}{2}x^2 + x + \ln|x| + C_2 & \text{if } x > 0 \end{cases}$