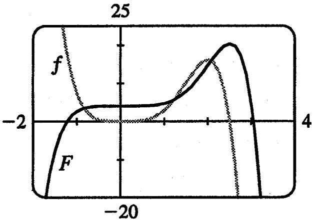
13. $f(x) = 5x^4 - 2x^5 \quad \Rightarrow \quad F(x) = 5 \cdot \dfrac{x^5}{5} - 2 \cdot \dfrac{x^6}{6} + C = x^5 - \frac{1}{3}x^6 + C.$

$F(0) = 4 \quad \Rightarrow \quad 0^5 - \frac{1}{3} \cdot 0^6 + C = 4 \quad \Rightarrow \quad C = 4$, so

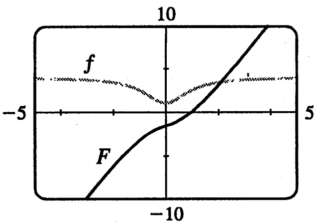
$F(x) = x^5 - \frac{1}{3}x^6 + 4$. The graph confirms our answer since $f(x) = 0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.



14. $f(x) = 4 - 3(1 + x^2)^{-1} = 4 - \dfrac{3}{1 + x^2} \quad \Rightarrow$

$F(x) = 4x - 3\tan^{-1} x + C$. $F(1) = 0 \quad \Rightarrow \quad 4 - 3\left(\frac{\pi}{4}\right) + C = 0 \quad \Rightarrow$

$C = \frac{3\pi}{4} - 4$, so $F(x) = 4x - 3\tan^{-1} x + \frac{3\pi}{4} - 4$. Note that f is positive and F is increasing on $\mathbb{R}$. Also, f has smaller values where the slopes of the tangent lines of F are smaller.



15. $f''(x) = 6x + 12x^2 \quad \Rightarrow \quad f'(x) = 6 \cdot \dfrac{x^2}{2} + 12 \cdot \dfrac{x^3}{3} + C = 3x^2 + 4x^3 + C \quad \Rightarrow$

$f(x) = 3 \cdot \dfrac{x^3}{3} + 4 \cdot \dfrac{x^4}{4} + Cx + D = x^3 + x^4 + Cx + D$ [C and D are just arbitrary constants]

16. $f''(x) = 2 + x^3 + x^6 \quad \Rightarrow \quad f'(x) = 2x + \frac{1}{4}x^4 + \frac{1}{7}x^7 + C \quad \Rightarrow \quad f(x) = x^2 + \frac{1}{20}x^5 + \frac{1}{56}x^8 + Cx + D$

17. $f''(x) = 1 + x^{4/5} \quad \Rightarrow \quad f'(x) = x + \frac{5}{9}x^{9/5} + C \quad \Rightarrow$

$f(x) = \frac{1}{2}x^2 + \frac{5}{9} \cdot \frac{5}{14}x^{14/5} + Cx + D = \frac{1}{2}x^2 + \frac{25}{126}x^{14/5} + Cx + D$

18. $f''(x) = \cos x \quad \Rightarrow \quad f'(x) = \sin x + C \quad \Rightarrow \quad f(x) = -\cos x + Cx + D$

19. $f'(x) = 3\cos x + 5\sin x \quad \Rightarrow \quad f(x) = 3\sin x - 5\cos x + C.$

$f(0) = 4 \quad \Rightarrow \quad -5 + C = 4 \quad \Rightarrow \quad C = 9$, so $f(x) = 3\sin x - 5\cos x + 9$.

20. $f'(x) = 4/\sqrt{1 - x^2} \quad \Rightarrow \quad f(x) = 4\sin^{-1} x + C$. $f\left(\frac{1}{2}\right) = 4\sin^{-1}\left(\frac{1}{2}\right) + C = 4 \cdot \frac{\pi}{6} + C$ and $f\left(\frac{1}{2}\right) = 1 \quad \Rightarrow$

$\frac{2\pi}{3} + C = 1 \quad \Rightarrow \quad C = 1 - \frac{2\pi}{3}$, so $f(x) = 4\sin^{-1} x + 1 - \frac{2\pi}{3}$.

21. $f''(x) = x \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^2 + C$. $f'(0) = 2 \quad \Rightarrow \quad C = 2$, so $f'(x) = \frac{1}{2}x^2 + 2 \quad \Rightarrow$

$f(x) = \frac{1}{6}x^3 + 2x + D$. $f(0) = -3 \quad \Rightarrow \quad D = -3$, so $f(x) = \frac{1}{6}x^3 + 2x - 3$.

22. $f''(x) = x + x^{1/2} \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$. $f'(1) = 2 \quad \Rightarrow \quad \frac{1}{2} + \frac{2}{3} + C = 2 \quad \Rightarrow \quad C = \frac{5}{6}$, so

$f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6} \quad \Rightarrow \quad f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D$. $f(1) = 1 \quad \Rightarrow \quad \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D = 1$

$\Rightarrow \quad D = -\frac{4}{15}$, so $f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$.

23. $f''(x) = x^{-2}, x > 0 \quad \Rightarrow \quad f'(x) = -1/x + C \quad \Rightarrow \quad f(x) = -\ln|x| + Cx + D = -\ln x + Cx + D$ (since $x > 0$). $f(1) = 0 \quad \Rightarrow \quad C + D = 0$ and $f(2) = 0 \quad \Rightarrow \quad -\ln 2 + 2C + D = 0 \quad \Rightarrow$

$-\ln 2 + 2C - C$ (since $D = -C$) $= 0 \quad \Rightarrow \quad -\ln 2 + C = 0 \quad \Rightarrow \quad C = \ln 2$ and $D = -\ln 2$. So

$f(x) = -\ln x + (\ln 2)x - \ln 2$.

24. $f''(x) = 3e^x + 5\sin x \quad \Rightarrow \quad f'(x) = 3e^x - 5\cos x + C$. $f'(0) = 2 \quad \Rightarrow \quad 3 - 5 + C = 2 \quad \Rightarrow \quad C = 4$, so

$f'(x) = 3e^x - 5\cos x + 4 \quad \Rightarrow \quad f(x) = 3e^x - 5\sin x + 4x + D$. $f(0) = 1 \quad \Rightarrow \quad 3 + D = 1 \quad \Rightarrow$

$D = -2$, so $f(x) = 3e^x - 5\sin x + 4x - 2$.