

35.  $v(t) = s'(t) = \sin t - \cos t \Rightarrow s(t) = -\cos t - \sin t + C$ .  $s(0) = -1 + C$  and  $s(0) = 0 \Rightarrow -1 + C = 0 \Rightarrow C = 1$ , so  $s(t) = -\cos t - \sin t + 1$ .
36.  $a(t) = v'(t) = 5 + 4t - 2t^2 \Rightarrow v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + C$ .  $v(0) = 3 \Rightarrow C = 3$ , so  $v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$ .  $v(t) = s'(t) \Rightarrow s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + D$ .  $s(0) = 10 \Rightarrow D = 10$ , so the particle's position after  $t$  seconds is given by  $s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + 10$ .
37. (a) We first observe that since the stone is dropped 450 m above the ground,  $v(0) = 0$  and  $s(0) = 450$ .  
 $v'(t) = a(t) = -9.8 \Rightarrow v(t) = -9.8t + C$ . Now  $v(0) = 0 \Rightarrow C = 0$ , so  $v(t) = -9.8t \Rightarrow s(t) = -4.9t^2 + D$ . Last,  $s(0) = 450 \Rightarrow D = 450 \Rightarrow s(t) = 450 - 4.9t^2$ .
- (b) The stone reaches the ground when  $s(t) = 0$ .  $450 - 4.9t^2 = 0 \Rightarrow t^2 = 450/4.9 \Rightarrow t_1 = \sqrt{450/4.9} \approx 9.58$  s.
- (c) The velocity with which the stone strikes the ground is  $v(t_1) = -9.8\sqrt{450/4.9} \approx -93.9$  m/s.
- (d) This is just reworking parts (a) and (b) with  $v(0) = -5$ . Using  $v(t) = -9.8t + C$ ,  $v(0) = -5 \Rightarrow 0 + C = -5 \Rightarrow v(t) = -9.8t - 5$ . So  $s(t) = -4.9t^2 - 5t + D$  and  $s(0) = 450 \Rightarrow D = 450 \Rightarrow s(t) = -4.9t^2 - 5t + 450$ . Solving  $s(t) = 0$  by using the quadratic formula gives us  $t = (5 \pm \sqrt{8845})/(-9.8) \Rightarrow t_1 \approx 9.09$  s.
38.  $v'(t) = a(t) = a \Rightarrow v(t) = at + C$  and  $v_0 = v(0) = C \Rightarrow v(t) = at + v_0 \Rightarrow s(t) = \frac{1}{2}at^2 + v_0t + D \Rightarrow s_0 = s(0) = D \Rightarrow s(t) = \frac{1}{2}at^2 + v_0t + s_0$
43. Using Exercise 38 with  $a = -32$ ,  $v_0 = 0$ , and  $s_0 = h$  (the height of the cliff), we know that the height at time  $t$  is  $s(t) = -16t^2 + h$ .  $v(t) = s'(t) = -32t$  and  $v(t) = -120 \Rightarrow -32t = -120 \Rightarrow t = 3.75$ , so  $0 = s(3.75) = -16(3.75)^2 + h \Rightarrow h = 16(3.75)^2 = 225$  ft.
44.  $v'(t) = a(t) = -40$ . The initial velocity is  $50$  mi/h =  $\frac{50 \cdot 5280}{3600} = \frac{220}{3}$  ft/s, so  $v(t) = -40t + \frac{220}{3}$ . The car stops when  $v(t) = 0 \Leftrightarrow t = \frac{220}{3 \cdot 40} = \frac{11}{6}$ . Since  $s(t) = -20t^2 + \frac{220}{3}t$ , the distance covered is  $s(\frac{11}{6}) = -20(\frac{11}{6})^2 + \frac{220}{3} \cdot \frac{11}{6} = \frac{605}{9} \approx 67.2$  ft.
45.  $a(t) = k$ , the initial velocity is  $30$  mi/h =  $30 \cdot \frac{5280}{3600} = 44$  ft/s, and the final velocity (after 5 seconds) is  $50$  mi/h =  $50 \cdot \frac{5280}{3600} = \frac{220}{3}$  ft/s. So  $v(t) = kt + C$  and  $v(0) = 44 \Rightarrow C = 44$ . Thus,  $v(t) = kt + 44 \Rightarrow v(5) = 5k + 44$ . But  $v(5) = \frac{220}{3}$ , so  $5k + 44 = \frac{220}{3} \Rightarrow 5k = \frac{88}{3} \Rightarrow k = \frac{88}{15} \approx 5.87$  ft/s<sup>2</sup>.
46.  $a(t) = -40 \Rightarrow v(t) = -40t + v_0$  where  $v_0$  is the car's speed (in ft/s) when the brakes were applied. The car stops when  $-40t + v_0 = 0 \Leftrightarrow t = \frac{1}{40}v_0$ . Now  $s(t) = \frac{1}{2}(-40)t^2 + v_0t = -20t^2 + v_0t$ . The car travels 160 ft in the time that it takes to stop, so  $s(\frac{1}{40}v_0) = 160 \Rightarrow 160 = -20(\frac{1}{40}v_0)^2 + v_0(\frac{1}{40}v_0) = \frac{1}{80}v_0^2 \Rightarrow v_0^2 = 12,800 \Rightarrow v_0 = 80\sqrt{2} \approx 113$  ft/s (about 77 mi/h).