

$$\begin{aligned}
 3. \text{ (a) } R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \quad [\Delta x = \frac{5-1}{4} = 1] \\
 &= f(x_1) \cdot 1 + f(x_2) \cdot 1 + f(x_3) \cdot 1 + f(x_4) \cdot 1 \\
 &= f(2) + f(3) + f(4) + f(5) \\
 &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} = 1.28\bar{3}
 \end{aligned}$$

Since  $f$  is *decreasing* on  $[1, 5]$ , an *underestimate* is obtained by using the *right endpoint approximation*,  $R_4$ .

$$\begin{aligned}
 \text{(b) } L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x \\
 &= f(1) + f(2) + f(3) + f(4) \\
 &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} = 2.08\bar{3}
 \end{aligned}$$

$L_4$  is an overestimate. Alternatively, we could just add the area of the leftmost rectangle and subtract the area of the rightmost; that is,

$$L_4 = R_4 + f(1) \cdot 1 - f(5) \cdot 1.$$

