

7. Let $u = x^2 + 3$. Then $du = 2x dx$, so $\int 2x(x^2 + 3)^4 dx = \int u^4 du = \frac{1}{5}u^5 + C = \frac{1}{5}(x^2 + 3)^5 + C$.

8. Let $u = x^2$. Then $du = 2x dx$, so $\int xe^{x^2} dx = \int e^u (\frac{1}{2} du) = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$.

9. Let $u = \ln x$. Then $du = \frac{dx}{x}$, so $\int \frac{(\ln x)^2}{x} dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$.

10. Let $u = 1 - x^4$. Then $du = -4x^3 dx$, so

$$\int x^3(1 - x^4)^5 dx = \int u^5(-\frac{1}{4} du) = -\frac{1}{4}(\frac{1}{6}u^6) + C = -\frac{1}{24}(1 - x^4)^6 + C$$

11. Let $u = x - 1$. Then $du = dx$, so $\int \sqrt{x-1} dx = \int u^{1/2} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(x-1)^{3/2} + C$.

12. Let $u = 2 - x$. Then $du = -dx$, so $\int (2-x)^6 dx = \int u^6(-du) = -\frac{1}{7}u^7 + C = -\frac{1}{7}(2-x)^7 + C$.

13. Let $u = 5 - 3x$. Then $du = -3 dx$, so $\int \frac{dx}{5-3x} = -\frac{1}{3} \int \frac{1}{u} du = -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|5-3x| + C$.

14. Let $u = x^2 + 1$. Then $du = 2x dx$, so

$$\int \frac{x}{x^2+1} dx = \int \frac{\frac{1}{2} du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2+1| + C = \frac{1}{2} \ln(x^2+1) + C \text{ [since } x^2+1 > 0 \text{]}$$

or $\ln \sqrt{x^2+1} + C$.

15. Let $u = 1 + x + 2x^2$. Then $du = (1 + 4x)dx$, so

$$\int \frac{1+4x}{\sqrt{1+x+2x^2}} dx = \int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{1+x+2x^2} + C$$

16. Let $u = 1 - t^3$. Then $du = -3t^2 dt$, so

$$\int t^2 \cos(1 - t^3) dt = \int \cos u(-\frac{1}{3} du) = -\frac{1}{3} \sin u + C = -\frac{1}{3} \sin(1 - t^3) + C$$

17. Let $u = t + 1$. Then $du = dt$, so $\int \frac{2}{(t+1)^6} dt = 2 \int u^{-6} du = -\frac{2}{5}u^{-5} + C = -\frac{2}{5(t+1)^5} + C$.

18. Let $u = 3 - 5y$. Then $du = -5 dy$, so

$$\int \sqrt[5]{3-5y} dy = \int u^{1/5}(-\frac{1}{5} du) = -\frac{1}{5} \cdot \frac{5}{6} u^{6/5} + C = -\frac{1}{6}(3-5y)^{6/5} + C$$

19. Let $u = 3\theta$. Then $du = 3 d\theta$, so $\int \sin 3\theta d\theta = \int \sin u(\frac{1}{3} du) = \frac{1}{3}(-\cos u) + C = -\frac{1}{3} \cos 3\theta + C$.

20. Let $u = \tan^{-1} x$. Then $du = \frac{dx}{1+x^2}$, so $\int \frac{\tan^{-1} x}{1+x^2} dx = \int u du = \frac{u^2}{2} + C = \frac{(\tan^{-1} x)^2}{2} + C$.

21. Let $u = 1 + e^x$. Then $du = e^x dx$, so $\int e^x \sqrt{1+e^x} dx = \int \sqrt{u} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+e^x)^{3/2} + C$.

Or: Let $u = \sqrt{1+e^x}$. Then $u^2 = 1+e^x$ and $2u du = e^x dx$, so

$$\int e^x \sqrt{1+e^x} dx = \int u \cdot 2u du = \frac{2}{3}u^3 + C = \frac{2}{3}(1+e^x)^{3/2} + C$$

22. Let $u = \sin x$. Then $du = \cos x dx$, so $\int \cot x dx = \int \frac{\cos x}{\sin x} dx = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$.