23. Let $u = \cos x$. Then $du = -\sin x\,dx$, so $\int \cos^4 x \sin x\,dx = \int u^4(-du) = -\frac{1}{5}u^5 + C = -\frac{1}{5}\cos^5 x + C$.

24. Let $u = \dfrac{\pi}{x}$. Then $du = -\dfrac{\pi}{x^2}\,dx$, so $\displaystyle\int \frac{\cos(\pi/x)}{x^2}\,dx = \int \cos u\left(-\frac{1}{\pi}\,du\right) = -\frac{1}{\pi}\sin u + C = -\frac{1}{\pi}\sin\frac{\pi}{x} + C$.

25. Let $u = \cot x$. Then $du = -\csc^2 x\,dx$, so

$$\int \sqrt{\cot x}\csc^2 x\,dx = \int \sqrt{u}\,(-du) = -\frac{u^{3/2}}{3/2} + C = -\tfrac{2}{3}(\cot x)^{3/2} + C$$

26. Let $u = \sin x$. Then $du = \cos x\,dx$, so $\int \cos x\,\cos(\sin x)dx = \int \cos u\,du = \sin u + C = \sin(\sin x) + C$.

27. $\displaystyle\int \frac{e^x + 1}{e^x}\,dx = \int \left(1 + e^{-x}\right)dx = x - e^{-x} + C$ [Substitute $u = -x$.]

28. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x + 1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x + 1) + C$.

29. Let $u = \sec x$. Then $du = \sec x\,\tan x\,dx$, so

$$\int \sec^3 x\,\tan x\,dx = \int \sec^2 x\,(\sec x\,\tan x)\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\sec^3 x + C$$

30. Let $u = \cos x$. Then $du = -\sin x\,dx$, so

$$\int \frac{\sin x}{1 + \cos^2 x}\,dx = \int \frac{-du}{1 + u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C$$

31. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\int \frac{1 + x}{1 + x^2}\,dx = \int \frac{1}{1 + x^2}\,dx + \int \frac{x}{1 + x^2}\,dx = \tan^{-1} x + \int \frac{\frac{1}{2}du}{u} = \tan^{-1} x + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1} x + \tfrac{1}{2}\ln\left|1 + x^2\right| + C = \tan^{-1} x + \tfrac{1}{2}\ln\left(1 + x^2\right) + C \text{ [since } 1 + x^2 > 0\text{]}.$$

32. Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1 + x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1 + u^2} = \tfrac{1}{2}\tan^{-1} u + C = \tfrac{1}{2}\tan^{-1}\left(x^2\right) + C$.