3. Let $u = x$, $dv = e^{2x}\,dx \;\Rightarrow\; du = dx$, $v = \frac{1}{2}e^{2x}$. Then by Equation 2,
$\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.$

4. Let $u = \ln x$, $dv = x^4\,dx \;\Rightarrow\; du = (1/x)\,dx$, $v = \frac{1}{5}x^5$. Then
$\int x^4 \ln x\,dx = \frac{1}{5}x^5 \ln x - \int \frac{1}{5}x^5(1/x)\,dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\left(\frac{1}{5}x^5\right) + C$
$= \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C$, or $\frac{1}{25}x^5(5 \ln x - 1) + C$.

5. Let $u = x$, $dv = \sin 4x\,dx \;\Rightarrow\; du = dx$, $v = -\frac{1}{4}\cos 4x$. Then by Equation 2,
$\int x \sin 4x\,dx = -\frac{1}{4}x \cos 4x - \int\left(-\frac{1}{4}\cos 4x\right)dx = -\frac{1}{4}x\cos 4x + \frac{1}{4}\left(\frac{1}{4}\sin 4x\right) + C$
$= -\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x + C.$

6. Let $u = \sin^{-1} x$, $dv = dx \;\Rightarrow\; du = \dfrac{dx}{\sqrt{1-x^2}}$, $v = x$. Then
$\int \sin^{-1} x\,dx = x \sin^{-1} x - \displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$. Setting $t = 1 - x^2$, we get $dt = -2x\,dx$, so
$-\displaystyle\int \frac{x\,dx}{\sqrt{1-x^2}} = -\int t^{-1/2}\left(-\tfrac{1}{2}\,dt\right) = \tfrac{1}{2}\left(2t^{1/2}\right) + C = t^{1/2} + C = \sqrt{1-x^2} + C$. Hence,
$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + C.$

7. First let $u = x^2$, $dv = \cos 3x\,dx \;\Rightarrow\; du = 2x\,dx$, $v = \frac{1}{3}\sin 3x$. Then by Equation 2, the original integral
$I = \int x^2 \cos 3x\,dx = \frac{1}{3}x^2 \sin 3x - \frac{2}{3}\int x \sin 3x\,dx$. To evaluate the last integral, we next let $U = x$,
$dV = \sin 3x\,dx \;\Rightarrow\; dU = dx$, $V = -\frac{1}{3}\cos 3x$. So
$\int x \sin 3x\,dx = -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\,dx = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C_1$. Substituting for $\int x\sin 3x\,dx$,
we get $I = \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C_1\right) = \frac{1}{3}x^2 \sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + C$,
where $C = -\frac{2}{3}C_1$.

8. First let $u = x^2$, $dv = \sin ax\,dx \;\Rightarrow\; du = 2x\,dx$, $v = -\dfrac{1}{a}\cos ax$. Then by Equation 2,
$I = \displaystyle\int x^2 \sin ax\,dx = -\frac{x^2}{a}\cos ax - \int\left(-\frac{1}{a}\right)\cos ax\,(2x\,dx) = -\frac{x^2}{a}\cos ax + \frac{2}{a}\int x\cos ax\,dx$. Next let
$U = x$, $dV = \cos ax\,dx \;\Rightarrow\; dU = dx$, $V = \dfrac{1}{a}\sin ax$. So
$\displaystyle\int x\cos ax\,dx = \frac{x}{a}\sin ax - \int \frac{1}{a}\sin ax\,dx = \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax + C_1$. Substituting for $\int x\cos ax\,dx$, we get
$I = -\dfrac{x^2}{a}\cos ax + \dfrac{2}{a}\left(\dfrac{x}{a}\sin ax + \dfrac{1}{a^2}\cos ax + C_1\right) = -\dfrac{x^2}{a}\cos ax + \dfrac{2x}{a^2}\sin ax + \dfrac{2}{a^3}\cos ax + C.$

9. First let $u = (\ln x)^2$, $dv = dx \;\Rightarrow\; du = 2\ln x \cdot \frac{1}{x}\,dx$, $v = x$. Then by Equation 2,
$I = \int (\ln x)^2\,dx = x(\ln x)^2 - 2\int x \ln x \cdot \frac{1}{x}\,dx = x(\ln x)^2 - 2\int \ln x\,dx$. Next let $U = \ln x$, $dV = dx \;\Rightarrow$
$dU = 1/x\,dx$, $V = x$ to get $\int \ln x\,dx = x\ln x - \int x\cdot(1/x)\,dx = x\ln x - \int dx = x\ln x - x + C_1$. Thus,
$I = x(\ln x)^2 - 2(x\ln x - x + C_1) = x(\ln x)^2 - 2x\ln x + 2x + C$, where $C = -2C_1$.

10. Let $u = t^3$, $dv = e^t\,dt \;\Rightarrow\; du = 3t^2\,dt$, $v = e^t$. Then $I = \int t^3 e^t\,dt = t^3e^t - \int 3t^2 e^t\,dt$. Integrate by parts twice more with $dv = e^t\,dt$.

$$\begin{aligned} I &= t^3e^t - \left(3t^2e^t - \textstyle\int 6te^t\,dt\right) = t^3e^t - 3t^2e^t + 6te^t - \textstyle\int 6e^t\,dt \\ &= t^3e^t - 3t^2e^t + 6te^t - 6e^t + C = \left(t^3 - 3t^2 + 6t - 6\right)e^t + C \end{aligned}$$

More generally, if $p(t)$ is a polynomial of degree n in t, then repeated integration by parts shows that
$\int p(t)\,e^t\,dt = \left[p(t) - p'(t) + p''(t) - p'''(t) + \cdots + (-1)^n p^{(n)}(t)\right]e^t + C.$

11. Let $u = \ln r$, $dv = r^3 dr \;\Rightarrow\; du = (1/r)\,dr$, $v = \frac{1}{4}r^4$. Then

$$\begin{aligned} \textstyle\int r^3 \ln r\,dr &= \tfrac{1}{4}r^4\ln r - \textstyle\int \tfrac{1}{4}r^4(1/r)\,dr = \tfrac{1}{4}r^4\ln r - \tfrac{1}{4}\textstyle\int r^3 dr = \tfrac{1}{4}r^4 \ln r - \tfrac{1}{4}\left(\tfrac{1}{4}r^4\right) + C \\ &= \tfrac{1}{4}r^4\ln r - \tfrac{1}{16}r^4 + C, \text{ or } \tfrac{1}{16}r^4(4\ln r - 1) + C. \end{aligned}$$