11. $xe^{-t}\dfrac{dx}{dt} = t$, $x(0) = 1$. $\int x\,dx = \int te^t\,dt \;\Rightarrow\; \frac{1}{2}x^2 = (t-1)\,e^t + C$ [by parts]. $x(0) = 1$, so $\frac{1}{2} = (0-1)e^0 + C$ and $C = \frac{3}{2}$. Thus, $\frac{1}{2}x^2 = (t-1)e^t + \frac{3}{2} \;\Rightarrow\; x^2 = 2(t-1)e^t + 3 \;\Rightarrow$ $x = \sqrt{2(t-1)e^t + 3}$ [use the positive square root since $x(0) = +1$].

12. $x + 2y\sqrt{x^2+1}\,\dfrac{dy}{dx} = 0$, $y(0) = 1$. $x\,dx + 2y\sqrt{x^2+1}\,dy = 0 \;\Rightarrow\; \displaystyle\int 2y\,dy = -\int \frac{x\,dx}{\sqrt{x^2+1}} \;\Rightarrow$ $y^2 = -\sqrt{x^2+1} + C$. $y(0) = 1 \;\Rightarrow\; 1 = -1 + C \;\Rightarrow\; C = 2$, so $y^2 = 2 - \sqrt{x^2+1}$ and $y = \sqrt{2 - \sqrt{x^2+1}}$.

13. $\dfrac{du}{dt} = \dfrac{2t + \sec^2 t}{2u}$, $u(0) = -5$. $\int 2u\,du = \int \left(2t + \sec^2 t\right) dt \;\Rightarrow\; u^2 = t^2 + \tan t + C$, where $[u(0)]^2 = 0^2 + \tan 0 + C \;\Rightarrow\; C = (-5)^2 = 25$. Therefore, $u^2 = t^2 + \tan t + 25$, so $u = \pm\sqrt{t^2 + \tan t + 25}$. Since $u(0) = -5$, we must have $u = -\sqrt{t^2 + \tan t + 25}$.

14. $\dfrac{dy}{dt} = te^y$, $y(1) = 0$. $\int e^{-y}\,dy = \int t\,dt \;\Rightarrow\; -e^{-y} = \frac{1}{2}t^2 + C$. Since $y(1) = 0$, $-e^0 = \frac{1}{2}\cdot 1^2 + C$. Therefore, $C = -1 - \frac{1}{2} = -\frac{3}{2}$ and $-e^{-y} = \frac{1}{2}t^2 - \frac{3}{2}$. So $e^{-y} = \frac{3}{2} - \frac{1}{2}t^2 = \dfrac{3-t^2}{2} \;\Rightarrow\; e^y = \dfrac{2}{3-t^2} \;\Rightarrow$ $y = \ln 2 - \ln\left(3 - t^2\right)$ for $|t| < \sqrt{3}$.

15. $\dfrac{dy}{dx} = 4x^3 y$, $y(0) = 7$. $\dfrac{dy}{y} = 4x^3\,dx$ [if $y \neq 0$] $\;\Rightarrow\; \displaystyle\int \frac{dy}{y} = \int 4x^3\,dx \;\Rightarrow\; \ln|y| = x^4 + C \;\Rightarrow$ $e^{\ln|y|} = e^{x^4 + C} \;\Rightarrow\; |y| = e^{x^4}e^C \;\Rightarrow\; y = Ae^{x^4}$; $y(0) = 7 \;\Rightarrow\; A = 7 \;\Rightarrow\; y = 7e^{x^4}$.

16. $\dfrac{dy}{dx} = \dfrac{y^2}{x^3}$, $y(1) = 1$. $\displaystyle\int \frac{dy}{y^2} = \int \frac{dx}{x^3} \;\Rightarrow\; -\frac{1}{y} = -\frac{1}{2x^2} + C$. $y(1) = 1 \;\Rightarrow\; -1 = -\frac{1}{2} + C \;\Rightarrow$ $C = -\frac{1}{2}$. So $\dfrac{1}{y} = \dfrac{1}{2x^2} + \dfrac{1}{2} = \dfrac{2 + 2x^2}{2\cdot 2x^2} \;\Rightarrow\; y = \dfrac{2x^2}{x^2+1}$.