1. A population of protozoa develops with a constant relative growth rate of 0.7944 per member per day. On day zero the population consists of two members. Find the population size after six days.

2. A common inhabitant of human intestines is the bacterium *Escherichia coli*. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 60 cells.
(a) Find the relative growth rate.
(b) Find an expression for the number of cells after t hours.
(c) Find the number of cells after 8 hours.
(d) Find the rate of growth after 8 hours.
(e) When will the population reach 20,000 cells?

3. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria.
(a) Find an expression for the number of bacteria after t hours.
(b) Find the number of bacteria after 4 hours.
(c) Find the rate of growth after 4 hours.
(d) When will the population reach 30,000?

4. A bacteria culture grows with constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 75,000.
(a) Find the initial population.
(b) Find an expression for the population after t hours.
(c) Find the number of cells after 5 hours.
(d) Find the rate of growth after 5 hours.
(e) When will the population reach 200,000?

5. The table gives estimates of the world population, in millions, from 1750 to 2000:

Year	Population	Year	Population
1750	790	1900	1650
1800	980	1950	2560
1850	1260	2000	6070

(a) Use the exponential model and the population figures for 1750 and 1800 to predict the world population in 1900 and 1950. Compare with the actual figures.
(b) Use the exponential model and the population figures for 1850 and 1900 to predict the world population in 1950. Compare with the actual population.
(c) Use the exponential model and the population figures for 1900 and 1950 to predict the world population in 2000. Compare with the actual population and try to explain the discrepancy.

6. The table gives the population of the United States, in millions, for the years 1900–2000.

Year	Population	Year	Population
1900	76	1960	179
1910	92	1970	203
1920	106	1980	227
1930	123	1990	250
1940	131	2000	275
1950	150		

(a) Use the exponential model and the census figures for 1900 and 1910 to predict the population in 2000. Compare with the actual figure and try to explain the discrepancy.
(b) Use the exponential model and the census figures for 1980 and 1990 to predict the population in 2000. Compare with the actual population. Then use this model to predict the population in the years 2010 and 2020.
(c) Draw a graph showing both of the exponential functions in parts (a) and (b) together with a plot of the actual population. Are these models reasonable ones?

7. Experiments show that if the chemical reaction

$$N_2O_5 \longrightarrow 2NO_2 + \tfrac{1}{2}O_2$$

takes place at 45 °C, the rate of reaction of dinitrogen pentoxide is proportional to its concentration as follows:

$$-\frac{d[N_2O_5]}{dt} = 0.0005[N_2O_5]$$

(See Example 4 in Section 3.3.)
(a) Find an expression for the concentration $[N_2O_5]$ after t seconds if the initial concentration is C.
(b) How long will the reaction take to reduce the concentration of N_2O_5 to 90% of its original value?

8. Bismuth-210 has a half-life of 5.0 days.
(a) A sample originally has a mass of 800 mg. Find a formula for the mass remaining after t days.
(b) Find the mass remaining after 30 days.
(c) When is the mass reduced to 1 mg?
(d) Sketch the graph of the mass function.

9. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.
(a) Find the mass that remains after t years.
(b) How much of the sample remains after 100 years?
(c) After how long will only 1 mg remain?

10. After 3 days a sample of radon-222 decayed to 58% of its original amount.
(a) What is the half-life of radon-222?
(b) How long would it take the sample to decay to 10% of its original amount?

11. Scientists can determine the age of ancient objects by a method called *radiocarbon dating*. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, ^{14}C, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates ^{14}C through food chains. When a plant or animal dies it stops replacing its carbon and the amount of ^{14}C begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially. A parchment fragment was discovered that had about 74% as much ^{14}C radioactivity as does plant material on Earth today. Estimate the age of the parchment.

12. A curve passes through the point (0, 5) and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

13. **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Suppose that a roast turkey is taken from an oven when its temperature has reached 185 °F and is placed on a table in a room where the temperature is 75 °F. If $u(t)$ is the temperature of the turkey after t minutes, then Newton's Law of Cooling implies that

$$\frac{du}{dt} = k(u - 75)$$

This could be solved as a separable differential equation. Another method is to make the change of variable $y = u - 75$.
(a) What initial-value problem does the new function y satisfy? What is the solution?
(b) If the temperature of the turkey is 150 °F after half an hour, what is the temperature after 45 min?
(c) When will the turkey have cooled to 100 °F?

STEWART
7.4