1. The relative growth rate is $\frac{1}{P}\frac{dP}{dt} = 0.7944$, so $\frac{dP}{dt} = 0.7944P$ and, by Theorem 2, $P(t) = P(0)e^{0.7944t} = 2e^{0.7944t}$. Thus, $P(6) = 2e^{0.7944(6)} \approx 234.99$ or about 235 members.

2. (a) By Theorem 2, $P(t) = P(0)e^{kt} = 60e^{kt}$. In 20 minutes ($\frac{1}{3}$ hour), there are 120 cells, so

$$P\left(\tfrac{1}{3}\right) = 60e^{k/3} = 120 \;\Rightarrow\; e^{k/3} = 2 \;\Rightarrow\; k/3 = \ln 2 \;\Rightarrow\; k = 3\ln 2 = \ln\left(2^3\right) = \ln 8.$$

(b) $P(t) = 60e^{(\ln 8)t} = 60 \cdot 8^t$

(c) $P(8) = 60 \cdot 8^8 = 60 \cdot 2^{24} = 1{,}006{,}632{,}960$

(d) $dP/dt = kP \;\Rightarrow\; P'(8) = kP(8) = (\ln 8)P(8) \approx 2.093$ billion cells/h

(e) $P(t) = 20{,}000 \;\Rightarrow\; 60 \cdot 8^t = 20{,}000 \;\Rightarrow\; 8^t = 1000/3 \;\Rightarrow\; t\ln 8 = \ln(1000/3) \;\Rightarrow$

$$t = \frac{\ln(1000/3)}{\ln 8} \approx 2.79 \text{ h}$$

3. (a) By Theorem 2, $y(t) = y(0)e^{kt} = 500e^{kt}$. Now $y(3) = 500e^{k(3)} = 8000 \;\Rightarrow\; e^{3k} = \frac{8000}{500} \;\Rightarrow$ $3k = \ln 16 \;\Rightarrow\; k = (\ln 16)/3$. So $y(t) = 500e^{(\ln 16)t/3} = 500 \cdot 16^{t/3}$

(b) $y(4) = 500 \cdot 16^{4/3} \approx 20{,}159$

(c) $dy/dt = ky \;\Rightarrow\; y'(4) = ky(4) = \frac{1}{3}\ln 16\left(500 \cdot 16^{4/3}\right)$ [from part (a)] $\approx 18{,}631$ cells/h

(d) $y(t) = 500 \cdot 16^{t/3} = 30{,}000 \;\Rightarrow\; 16^{t/3} = 60 \;\Rightarrow\; \frac{1}{3}t\ln 16 = \ln 60 \;\Rightarrow\; t = 3(\ln 60)/(\ln 16) \approx 4.4$ h

4. (a) $y(t) = y(0)e^{kt} \;\Rightarrow\; y(2) = y(0)e^{2k} = 600$, $y(8) = y(0)e^{8k} = 75{,}000$. Dividing these equations, we get $e^{8k}/e^{2k} = 75{,}000/600 \;\Rightarrow\; e^{6k} = 125 \;\Rightarrow\; 6k = \ln 125 = \ln 5^3 = 3\ln 5 \;\Rightarrow\; k = \frac{3}{6}\ln 5 = \frac{1}{2}\ln 5$. Thus, $y(0) = 600/e^{2k} = 600/e^{\ln 5} = \frac{600}{5} = 120$.

(b) $y(t) = y(0)e^{kt} = 120e^{(\ln 5)t/2}$ or $y = 120 \cdot 5^{t/2}$

(c) $y(5) = 120 \cdot 5^{5/2} = 120 \cdot 25\sqrt{5} = 3000\sqrt{5} \approx 6708$ bacteria.

(d) $y(t) = 120 \cdot 5^{t/2} \;\Rightarrow\; y'(t) = 120 \cdot 5^{t/2} \cdot \ln 5 \cdot \frac{1}{2} = 60 \cdot \ln 5 \cdot 5^{t/2}$.
$y'(5) = 60 \cdot \ln 5 \cdot 5^{5/2} = 60 \cdot \ln 5 \cdot 25\sqrt{5} \approx 5398$ bacteria/hour.

(e) $y(t) = 200{,}000 \;\Leftrightarrow\; 120e^{(\ln 5)t/2} = 200{,}000 \;\Leftrightarrow\; e^{(\ln 5)t/2} = \frac{5000}{3} \;\Leftrightarrow\; (\ln 5)t/2 = \ln\frac{5000}{3} \;\Leftrightarrow$
$t = \left(2\ln\frac{5000}{3}\right)/\ln 5 \approx 9.2$ h.