7. (a) If $y = [N_2O_5]$ then by Theorem 2, $\frac{dy}{dt} = -0.0005y \Rightarrow y(t) = y(0)e^{-0.0005t} = Ce^{-0.0005t}$.

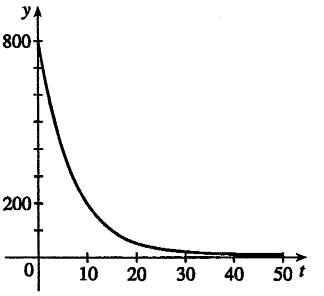
(b) $y(t) = Ce^{-0.0005t} = 0.9C \Rightarrow e^{-0.0005t} = 0.9 \Rightarrow -0.0005t = \ln 0.9 \Rightarrow$
$t = -2000 \ln 0.9 \approx 211$ s

8. (a) The mass remaining after t days is
$y(t) = y(0)e^{kt} = 800e^{kt}$. Since the half-life is 5.0 days,
$y(5) = 800e^{5k} = 400 \Rightarrow e^{5k} = \frac{1}{2} \Rightarrow$
$5k = \ln \frac{1}{2} \Rightarrow k = -(\ln 2)/5$, so
$y(t) = 800e^{-(\ln 2)t/5} = 800 \cdot 2^{-t/5}$.

(b) $y(30) = 800 \cdot 2^{-30/5} = 12.5$ mg

(c) $800e^{-(\ln 2)t/5} = 1 \Leftrightarrow -(\ln 2)\frac{t}{5} = \ln \frac{1}{800} = -\ln 800$
$\Leftrightarrow t = 5\frac{\ln 800}{\ln 2} \approx 48$ days

(d)



9. (a) If $y(t)$ is the mass remaining after t days, then $y(t) = y(0)e^{kt} = 100e^{kt}$. $y(30) = 100e^{30k} = \frac{1}{2}(100) \Rightarrow$
$e^{30k} = \frac{1}{2} \Rightarrow k = -(\ln 2)/30 \Rightarrow y(t) = 100e^{-(\ln 2)t/30} = 100 \cdot 2^{-t/30}$

(b) $y(100) = 100 \cdot 2^{-100/30} \approx 9.92$ mg

(c) $100e^{-(\ln 2)t/30} = 1 \Rightarrow -(\ln 2)t/30 = \ln \frac{1}{100} \Rightarrow t = -30\frac{\ln 0.01}{\ln 2} \approx 199.3$ years

10. (a) If $y(t)$ is the mass after t days and $y(0) = A$, then $y(t) = Ae^{kt}$. $y(3) = Ae^{3k} = 0.58A \Rightarrow$
$e^{3k} = 0.58 \Rightarrow 3k = \ln 0.58 \Rightarrow k = \frac{1}{3}\ln 0.58$. Then $Ae^{\ln(0.58)t/3} = \frac{1}{2}A \Leftrightarrow$
$\ln e^{\ln(0.58)t/3} = \ln \frac{1}{2} \Leftrightarrow \frac{\ln(0.58)t}{3} = \ln \frac{1}{2}$, so the half-life is $t = -\frac{3\ln 2}{\ln 0.58} \approx 3.82$ days.

(b) $Ae^{\ln(0.58)t/3} = 0.10A \Leftrightarrow \frac{\ln(0.58)t}{3} = \ln \frac{1}{10} \Leftrightarrow t = -\frac{3\ln 10}{\ln 0.58} \approx 12.68$ days

11. Let $y(t)$ be the level of radioactivity. Thus, $y(t) = y(0)e^{-kt}$ and k is determined by using the half-life:
$y(5730) = \frac{1}{2}y(0) \Rightarrow y(0)e^{-k(5730)} = \frac{1}{2}y(0) \Rightarrow e^{-5730k} = \frac{1}{2} \Rightarrow$
$-5730k = \ln \frac{1}{2} \Rightarrow k = -\frac{\ln \frac{1}{2}}{5730} = \frac{\ln 2}{5730}$. If 74% of the ^{14}C remains, then we know that $y(t) = 0.74y(0)$
$\Rightarrow 0.74 = e^{-t(\ln 2)/5730} \Rightarrow \ln 0.74 = -\frac{t\ln 2}{5730} \Rightarrow t = -\frac{5730(\ln 0.74)}{\ln 2} \approx 2489 \approx 2500$ years.

12. From the information given, we know that $\frac{dy}{dx} = 2y \Rightarrow y = Ce^{2x}$ by Theorem 2. To calculate C we use the point $(0, 5)$: $5 = Ce^{2(0)} \Rightarrow C = 5$. Thus, the equation of the curve is $y = 5e^{2x}$.

13. (a) If $y = u - 75$, $u(0) = 185 \Rightarrow y(0) = 185 - 75 = 110$, and the initial-value problem is $dy/dt = ky$ with $y(0) = 110$. So the solution is $y(t) = 110e^{kt}$.

(b) $y(30) = 110e^{30k} = 150 - 75 \Rightarrow e^{30k} = \frac{75}{110} = \frac{15}{22} \Rightarrow k = \frac{1}{30}\ln \frac{15}{22}$, so $y(t) = 110e^{\frac{1}{30}t\ln\left(\frac{15}{22}\right)}$ and
$y(45) = 110e^{\frac{45}{30}\ln\left(\frac{15}{22}\right)} \approx 62\,°$F. Thus, $u(45) \approx 62 + 75 = 137\,°$F.

(c) $u(t) = 100 \Rightarrow y(t) = 25$. $y(t) = 110e^{\frac{1}{30}t\ln\left(\frac{15}{22}\right)} = 25 \Rightarrow e^{\frac{1}{30}t\ln\left(\frac{15}{22}\right)} = \frac{25}{110} \Rightarrow$
$\frac{1}{30}t\ln \frac{15}{22} = \ln \frac{25}{110} \Rightarrow t = \frac{30\ln \frac{25}{110}}{\ln \frac{15}{22}} \approx 116$ min.