14. (a) Let $y(t)$ = temperature after t minutes. Newton's Law of Cooling implies that $\frac{dy}{dt} = k(y-5)$. Let $u(t) = y(t) - 5$. Then $\frac{du}{dt} = ku$, so $u(t) = u(0)e^{kt} = (20-5)e^{kt} \;\Rightarrow\; y(t) = 5 + 15e^{kt} \;\Rightarrow$ $y(1) = 5 + 15e^{k} = 12 \;\Rightarrow\; e^{k} = \frac{7}{15} \;\Rightarrow\; k = \ln\frac{7}{15}$, so $y(t) = 5 + 15e^{\ln(7/15)t}$ and $y(2) = 5 + 15e^{2\ln(7/15)} \approx 8.3\ ^\circ\text{C}$.

(b) $5 + 15e^{\ln(7/15)t} = 6$ when $e^{\ln(7/15)t} = \frac{1}{15} \;\Rightarrow\; \ln\left(\frac{7}{15}\right)t = \ln\frac{1}{15} \;\Rightarrow\; t = \dfrac{\ln\frac{1}{15}}{\ln\frac{7}{15}} \approx 3.6$ min.

15. (a) Let $P(h)$ be the pressure at altitude h. Then $dP/dh = kP \;\Rightarrow\; P(h) = P(0)e^{kh} = 101.3e^{kh}$. $P(1000) = 101.3e^{1000k} = 87.14 \;\Rightarrow\; 1000k = \ln\left(\frac{87.14}{101.3}\right) \;\Rightarrow$ $k = \frac{1}{1000}\ln\left(\frac{87.14}{101.3}\right) \;\Rightarrow\; P(h) = 101.3\,e^{\frac{1}{1000}h\ln\left(\frac{87.14}{101.3}\right)}$, so $P(3000) = 101.3e^{3\ln\left(\frac{87.14}{101.3}\right)} \approx 64.5$ kPa.

(b) $P(6187) = 101.3\,e^{\frac{6187}{1000}\ln\left(\frac{87.14}{101.3}\right)} \approx 39.9$ kPa

16. (a) Using $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$ with $A_0 = 500$, $r = 0.14$, and $t = 2$, we have:

(i) Annually: $n = 1$; $A = 500\left(1 + \frac{0.14}{1}\right)^{1\cdot 2} = \$649.80$

(ii) Quarterly: $n = 4$; $A = 500\left(1 + \frac{0.14}{4}\right)^{4\cdot 2} = \$658.40$

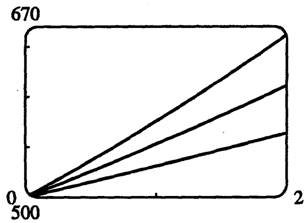
(iii) Monthly: $n = 12$; $A = 500\left(1 + \frac{0.14}{12}\right)^{12\cdot 2} = \$660.49$

(iv) Daily: $n = 365$; $A = 500\left(1 + \frac{0.14}{365}\right)^{365\cdot 2} = \$661.53$

(v) Hourly: $n = 365\cdot 24$; $A = 500\left(1 + \frac{0.14}{365\cdot 24}\right)^{365\cdot 24\cdot 2} = \$661.56$

(vi) Continuously: $A = 500e^{(0.14)2} = \$661.56$

(b)



$A_{0.14}(2) = \$661.56$,
$A_{0.10}(2) = \$610.70$, and
$A_{0.06}(2) = \$563.75$.

17. (a) Using $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$ with $A_0 = 3000$, $r = 0.05$, and $t = 5$, we have:

(i) Annually: $n = 1$; $A = 3000\left(1 + \frac{0.05}{1}\right)^{1\cdot 5} = \$3828.84$

(ii) Semiannually: $n = 2$; $A = 3000\left(1 + \frac{0.05}{2}\right)^{2\cdot 5} = \$3840.25$

(iii) Monthly: $n = 12$; $A = 3000\left(1 + \frac{0.05}{12}\right)^{12\cdot 5} = \$3850.08$

(iv) Weekly: $n = 52$; $A = 3000\left(1 + \frac{0.05}{52}\right)^{52\cdot 5} = \$3851.61$

(v) Daily: $n = 365$; $A = 3000\left(1 + \frac{0.05}{365}\right)^{365\cdot 5} = \$3852.01$

(vi) Continuously: $A = 3000e^{(0.05)5} = \$3852.08$

(b) $dA/dt = 0.05A$ and $A(0) = 3000$.

18. (a) $A_0e^{0.06t} = 2A_0 \;\Leftrightarrow\; e^{0.06t} = 2 \;\Leftrightarrow\; 0.06t = \ln 2 \;\Leftrightarrow\; t = \frac{50}{3}\ln 2 \approx 11.55$, so the investment will double in about 11.55 years.

(b) The annual interest rate in $A = A_0(1+r)^t$ is r. From part (a), we have $A = A_0e^{0.06t}$. These amounts must be equal, so $(1+r)^t = e^{0.06t} \;\Rightarrow\; 1 + r = e^{0.06} \;\Rightarrow\; r = e^{0.06} - 1 \approx 0.0618 = 6.18\%$, which is the equivalent annual interest rate.