14. A thermometer is taken from a room where the temperature is 20 °C to the outdoors, where the temperature is 5 °C. After one minute the thermometer reads 12 °C. Use Newton's Law of Cooling to answer the following questions.

(a) What will the reading on the thermometer be after one more minute?

(b) When will the thermometer read 6 °C?

15. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At 15 °C the pressure is 101.3 kPa at sea level and 87.14 kPa at $h = 1000$ m.

(a) What is the pressure at an altitude of 3000 m?

(b) What is the pressure at the top of Mount McKinley, at an altitude of 6187 m?

16. (a) If \$500 is borrowed at 14% interest, find the amounts due at the end of 2 years if the interest is compounded

(i) annually, (ii) quarterly, (iii) monthly, (iv) daily, (v) hourly, and (vi) continuously.

(b) Suppose \$500 is borrowed and the interest is compounded continuously. If $A(t)$ is the amount due after t years, where $0 \leqslant t \leqslant 2$, graph $A(t)$ for each of the interest rates 14%, 10%, and 6% on a common screen.

17. (a) If \$3000 is invested at 5% interest, find the value of the investment at the end of 5 years if the interest is compounded (i) annually, (ii) semiannually, (iii) monthly, (iv) weekly, (v) daily, and (vi) continuously.

(b) If $A(t)$ is the amount of the investment at time t for the case of continuous compounding, write a differential equation and an initial condition satisfied by $A(t)$.

18. (a) How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

(b) What is the equivalent annual interest rate?

19. Consider a population $P = P(t)$ with constant relative birth and death rates α and β, respectively, and a constant emigration rate m, where α, β, and m are positive constants. Assume that $\alpha > \beta$. Then the rate of change of the population at time t is modeled by the differential equation

$$\frac{dP}{dt} = kP - m \qquad \text{where } k = \alpha - \beta$$

(a) Find the solution of this equation that satisfies the initial condition $P(0) = P_0$.

(b) What condition on m will lead to an exponential expansion of the population?

(c) What condition on m will result in a constant population? A population decline?

(d) In 1847, the population of Ireland was about 8 million and the difference between the relative birth and death rates was 1.6% of the population. Because of the potato famine in the 1840s and 1850s, about 210,000 inhabitants per year emigrated from Ireland. Was the population expanding or declining at that time?

20. Let c be a positive number. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

where k is a positive constant, is called a *doomsday equation* because the exponent in the expression ky^{1+c} is larger than that for natural growth (that is, ky).

(a) Determine the solution that satisfies the initial condition $y(0) = y_0$.

(b) Show that there is a finite time $t = T$ such that $\lim_{t \to T^-} y(t) = \infty$.

(c) An especially prolific breed of rabbits has the growth term $ky^{1.01}$. If 2 such rabbits breed initially and the warren has 16 rabbits after three months, then when is doomsday?