Exercises 4.8

1. By the Horizontal Line Test, f is not one-to-one.

3. By the Horizontal Line Test, f is one-to-one.

5. By the Horizontal Line Test, f is not one-to-one.

17. (a) $f(2) = 7$. Since f is one-to-one, $f^{-1}(7) = 2$.
(b) $f^{-1}(3) = -1$. Since f is one-to-one, $f(-1) = 3$.

19. $f(x) = 5 - 2x$. Since f is one-to-one and $f(1) = 5 - 2(1) = 3$, then $f^{-1}(3) = 1$. (Find 1 by solving the equation $5 - 2x = 3$.)

31. $f(x) = 2x + 1$. $y = 2x + 1 \quad \Leftrightarrow \quad 2x = y - 1 \quad \Leftrightarrow \quad x = \frac{1}{2}(y - 1)$. So $f^{-1}(x) = \frac{1}{2}(x - 1)$.

33. $f(x) = 4x + 7$. $y = 4x + 7 \quad \Leftrightarrow \quad 4x = y - 7 \quad \Leftrightarrow \quad x = \frac{1}{4}(y - 7)$. So $f^{-1}(x) = \frac{1}{4}(x - 7)$.

35. $f(x) = \dfrac{x}{2}$. $y = \dfrac{x}{2} \quad \Leftrightarrow \quad x = 2y$. So $f^{-1}(x) = 2x$.

37. $f(x) = \dfrac{1}{x+2}$. $y = \dfrac{1}{x+2} \quad \Leftrightarrow \quad x + 2 = \dfrac{1}{y} \quad \Leftrightarrow \quad x = \dfrac{1}{y} - 2$. So $f^{-1}(x) = \dfrac{1}{x} - 2$.

39. $f(x) = \dfrac{1+3x}{5-2x}$. $y = \dfrac{1+3x}{5-2x} \quad \Leftrightarrow \quad y(5 - 2x) = 1 + 3x \quad \Leftrightarrow \quad 5y - 2xy = 1 + 3x \quad \Leftrightarrow$
$3x + 2xy = 5y - 1 \quad \Leftrightarrow \quad x(3 + 2y) = 5y - 1 \quad \Leftrightarrow \quad x = \dfrac{5y-1}{2y+3}$. So $f^{-1}(x) = \dfrac{5x-1}{2x+3}$.

41. $f(x) = \sqrt{2 + 5x}, x \geq -\frac{2}{5}$. $y = \sqrt{2 + 5x}, y \geq 0 \quad \Leftrightarrow \quad y^2 = 2 + 5x \quad \Leftrightarrow \quad 5x = y^2 - 2 \quad \Leftrightarrow$
$x = \frac{1}{5}(y^2 - 2)$ and $y \geq 0$. So $f^{-1}(x) = \frac{1}{5}(x^2 - 2), x \geq 0$.

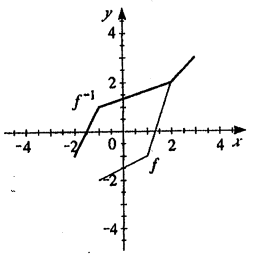
43. $f(x) = 4 - x^2,\ x \geq 0$. $y = 4 - x^2 \quad \Leftrightarrow \quad x^2 = 4 - y \quad \Leftrightarrow \quad x = \sqrt{4 - y}$. So
$f^{-1}(x) = \sqrt{4 - x}$. Note: $x \geq 0 \quad \Rightarrow \quad f(x) \leq 4$.

45. $f(x) = 4 + \sqrt[3]{x}$. $y = 4 + \sqrt[3]{x} \quad \Leftrightarrow \quad \sqrt[3]{x} = y - 4 \quad \Leftrightarrow \quad x = (y - 4)^3$. So $f^{-1}(x) = (x - 4)^3$.

47. $f(x) = 1 + \sqrt{1 + x}$. $y = 1 + \sqrt{1 + x}, y \geq 1$. $\Leftrightarrow \quad \sqrt{1 + x} = y - 1 \quad \Leftrightarrow \quad 1 + x = (y - 1)^2$
$\Leftrightarrow \quad x = (y - 1)^2 - 1 = y^2 - 2y$. So $f^{-1}(x) = x^2 - 2x, x \geq 1$.

49. $f(x) = x^4, x \geq 0$. $y = x^4, y \geq 0 \quad \Leftrightarrow \quad x = \sqrt[4]{y}$. So $f^{-1}(x) = \sqrt[4]{x}, x \geq 0$.

65.



67. $f(x) = 7 + 2x$. $y = 7 + 2x \quad \Leftrightarrow \quad 2x = y - 7 \quad \Leftrightarrow \quad x = \dfrac{y-7}{2}$. So $f^{-1}(x) = \dfrac{x-7}{2}$. f^{-1} is the number of toppings on a pizza that costs $f(x)$ dollars.