

49.  $128 + 16t - 16t^2 \geq 32 \Leftrightarrow -16t^2 + 16t + 96 \geq 0 \Leftrightarrow -16(t^2 - t - 6) \geq 0 \Leftrightarrow -16(t - 3)(t + 2) \geq 0$ . The expression on the left of the inequality changes sign at  $x = -2$ , at  $t = 3$ , and at  $t = -2$ . However,  $t \geq 0$ , so the only endpoint is  $t = 3$ . Thus we check the intervals in the following table.

Interval	(0, 3)	(3, $\infty$ )
Sign of $-16$	-	-
Sign of $t - 3$	-	+
Sign of $t + 2$	+	+
Sign of $-16(t - 3)(t + 2)$	+	-

So  $0 \leq t \leq 3$ .

51.  $\frac{600,000}{x^2 + 300} < 500 \Leftrightarrow 600,000 < 500(x^2 + 300)$  (Note that  $x^2 + 300 \geq 300 > 0$ , so we can multiply both sides by the denominator and not worry that we might be multiplying both sides by a negative number or by zero.)  $1200 < x^2 + 300 \Leftrightarrow 0 < x^2 - 900 \Leftrightarrow 0 < (x - 30)(x + 30)$ . The expression in the inequality changes sign at  $x = 30$  and  $x = -30$ . However, since  $x$  represents distance, we must have  $x > 0$ .

Interval	(0, 30)	(30, $\infty$ )
Sign of $x - 30$	-	+
Sign of $x + 30$	+	+
Sign of $(x - 30)(x + 30)$	-	+

So  $x > 30$  and you must stand at least 30 meters from the center of the fire.

53.  $240 > v + \frac{v^2}{20} \Leftrightarrow \frac{1}{20}v^2 + v - 240 < 0 \Leftrightarrow (\frac{1}{20}v - 3)(v + 80) < 0$ . The expression in the inequality changes sign at  $v = 60$  and  $v = -80$ . However, since  $v$  represents the speed, we must have  $v \geq 0$ .

Interval	(0, 60)	(60, $\infty$ )
Sign of $\frac{1}{20}v - 3$	-	+
Sign of $v + 80$	+	+
Sign of $(\frac{1}{20}v - 3)(v + 80)$	-	+

So Kerry must drive less than 60 mph.

55. Let  $x$  be the length of the garden. Using the fact that the perimeter is 120 ft, we must have  $2x + 2\text{width} = 120 \Leftrightarrow \text{width} = 60 - x$ . Now since the area must be at least 800 ft<sup>2</sup>, we have  $800 < x(60 - x) \Leftrightarrow 800 < 60x - x^2 \Leftrightarrow x^2 - 60x + 800 < 0 \Leftrightarrow (x - 20)(x - 40) < 0$ . The expression in the inequality changes sign at  $x = 20$  and  $x = 40$ . However, since  $x$  represents length, we must have  $x > 0$ .

Interval	(0, 20)	(20, 40)	(40, $\infty$ )
Sign of $x - 20$	-	+	+
Sign of $x - 40$	-	-	+
Sign of $(x - 20)(x - 40)$	+	-	+

The length of the garden should be between 20 and 40 feet.