

50. $0.004 \leq \frac{4,000,000}{d^2} \leq 0.01$. Since $d^2 \geq 0$ and $d \neq 0$, we can multiply each expression by d^2 to obtain $0.004d^2 \leq 4,000,000 \leq 0.01d^2$. Solving each pair, we have $0.004d^2 \leq 4,000,000 \Leftrightarrow d^2 \leq 1,000,000,000 \Rightarrow d \leq 10,000\sqrt{10}$ (recall that d represents distance, so it is always nonnegative). Solving $4,000,000 \leq 0.01d^2 \Leftrightarrow 400,000,000 \leq d^2 \Rightarrow 20,000 \leq d$. Putting these together, we have $20,000 \leq d \leq 10,000\sqrt{10}$.

52. Solve $30 \leq 10 + 0.9v - 0.01v^2$ for $10 \leq v \leq 75$. We have $30 \leq 10 + 0.9v - 0.01v^2 \Leftrightarrow 0.01v^2 - 0.9v + 20 \leq 0 \Leftrightarrow (0.1v - 4)(0.1v - 5) \leq 0$. The possible endpoints are $0.1v - 4 = 0 \Leftrightarrow 0.1v = 4 \Leftrightarrow v = 40$ and $0.1v - 5 = 0 \Leftrightarrow 0.1v = 5 \Leftrightarrow v = 50$.

Interval	(10, 40)	(40, 50)	(50, 75)
Sign of $0.1v - 4$	-	+	+
Sign of $0.1v - 5$	-	-	+
Sign of $(0.1v - 4)(0.1v - 5)$	+	-	+

Thus he must drive between 40 and 50 mi/h.

54. Solve $2400 \leq 20x - (2000 + 8x + 0.0025x^2) \Leftrightarrow 2400 \leq 20x - 2000 - 8x - 0.0025x^2 \Leftrightarrow 0.0025x^2 - 12x + 4400 \leq 0 \Leftrightarrow (0.0025x - 1)(x - 4400) \leq 0$. The expression on the left of the inequality changes sign when $x = 400$ and $x = 4400$. Since the manufacturer can only sell positive units, we check the intervals in the following table.

Interval	(0, 400)	(400, 4400)	(4400, ∞)
Sign of $0.0025x - 1$	-	+	+
Sign of $x - 4400$	-	-	+
Sign of $(0.0025x - 1)(x - 4400)$	+	-	+

So the manufacturer must sell between 400 and 4400 units to enjoy a profit of at least \$2400.

56. Let n be the number of people in the group. Then the bus fare is $\frac{360}{n}$, and the cost of the theater tickets is $30 - 0.25n$. We want the total cost to be less than \$39 per person, that is, $\frac{360}{n} + (30 - 0.25n) < 39$. If we multiply this inequality by n , we will not change the direction of the inequality; n is positive since it represents the number of people. So we get $360 + n(30 - 0.25n) < 39n \Leftrightarrow -0.25n^2 + 30n + 360 < 39n \Leftrightarrow -0.25n^2 - 9n + 360 < 0 \Leftrightarrow (0.25n + 15)(-n + 24)$. The expression on the left of the inequality changes sign when $n = -60$ and $n = 24$. Since $n > 0$, we check the intervals in the following table.

Interval	(0, 24)	(24, ∞)
Sign of $0.25n + 15$	+	+
Sign of $-n + 24$	+	-
Sign of $(0.25n + 15)(-n + 24)$	+	-

So the group must have more than 24 people in order that the cost of the theater tour is less than \$39.