

EXERCISE SET 4.5

- Find a number in the closed interval $[\frac{1}{2}, \frac{3}{2}]$ such that the sum of the number and its reciprocal is
 - as small as possible
 - as large as possible.
- How should two nonnegative numbers be chosen so that their sum is 1 and the sum of their squares is
 - as large as possible
 - as small as possible?
- A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.
- A field has boundary a right triangle with hypotenuse along a straight stream. A fence bounds the other two sides of the field. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.
- A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$6000?
- A rectangle is to be inscribed in a right triangle having sides of length 6 in, 8 in, and 10 in. Find the dimensions of the rectangle with greatest area assuming the rectangle is positioned as in Figure Ex-6.
- Solve the problem in Exercise 6 assuming the rectangle is positioned as in Figure Ex-7.

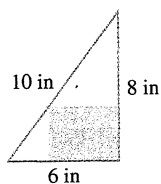


Figure Ex-6

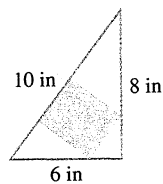


Figure Ex-7

- A rectangle has its two lower corners on the x -axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?
- Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10.

- What is the largest possible area of a region in the plane that is contained in both the rectangle with corners at $(\pm 8, \pm 10)$ and a square whose sides are parallel to the coordinate axes and whose lower left corner is on the line $y = -4x$?
- A rectangular area of 3200 ft² is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the rectangle of least cost.
- Show that among all rectangles with perimeter p , the square has the maximum area.
- Show that among all rectangles with area A , the square has the minimum perimeter.
- A wire of length 12 in can be bent into a circle, bent into a square, or cut into two pieces to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be
 - a maximum
 - a minimum?
- A field in the shape of an isosceles triangle is to be bounded by a fence on the two equal sides of the triangle, and by a straight stream on the third side. Find the dimensions of the field of largest area that can be enclosed by 300 yards of fence.
- A church window consisting of a rectangle topped by a semi-circle is to have a perimeter p . Find the radius of the semi-circle if the area of the window is to be maximum.
- A box with a square base is taller than it is wide. In order to send the box through the U.S. mail, the height of the box and the perimeter of the base can sum to no more than 108 in. What is the maximum volume for such a box?
- A box with a square base is wider than it is tall. In order to send the box through the U.S. mail, the width of the box and the perimeter of one of the (nonsquare) sides of the box can sum to no more than 108 in. What is the maximum volume for such a box?
- An open box is to be made from a 3-ft by 8-ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have.
- A closed rectangular container with a square base is to have a volume of 2250 in³. The material for the top and bottom of the container will cost \$2 per in², and the material for the sides will cost \$3 per in². Find the dimensions of the container of least cost.

- A closed rectangular container with a square base is to have a volume of 2000 cm³. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container of least cost.
- A container with square base, vertical sides, and open top is to be made from 1000 ft² of material. Find the dimensions of the container with greatest volume.
- A rectangular container with two square sides and an open top is to have a volume of V cubic units. Find the dimensions of the container with minimum surface area.
- Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius R .
- Find the dimensions of the right circular cylinder of greatest surface area that can be inscribed in a sphere of radius R .
- Show that the right circular cylinder of greatest volume that can be inscribed in a right circular cone has volume that is $\frac{4}{9}$ the volume of the cone (Figure Ex-26).

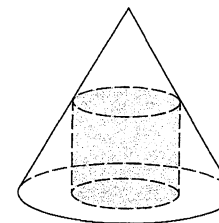


Figure Ex-26

- A closed, cylindrical can is to have a volume of V cubic units. Show that the can of minimum surface area is achieved when the height is equal to the diameter of the base.
- A closed cylindrical can is to have a surface area of S square units. Show that the can of maximum volume is achieved when the height is equal to the diameter of the base.
- A cylindrical can, open at the top, is to hold 500 cm³ of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.

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