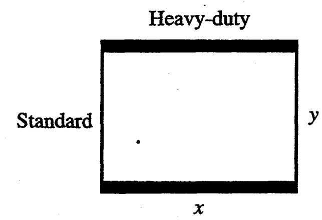
5. Let x and y be the dimensions shown in the figure and A the area, then $A = xy$ subject to the cost condition $3(2x) + 2(2y) = 6000$, or $y = 1500 - 3x/2$. Thus $A = x(1500 - 3x/2) = 1500x - 3x^2/2$ for x in $[0, 1000]$. $dA/dx = 1500 - 3x$, $dA/dx = 0$ when $x = 500$. If $x = 0$ or 1000 then $A = 0$, if $x = 500$ then $A = 375{,}000$ so the area is greatest when $x = 500$ ft and (from $y = 1500 - 3x/2$) when $y = 750$ ft.



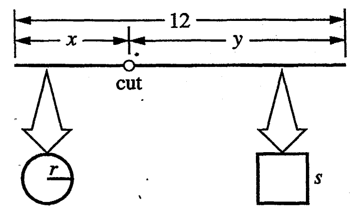
14. With x, y, r, and s as shown in the figure, the sum of the enclosed areas is $A = \pi r^2 + s^2$ where $r = \dfrac{x}{2\pi}$ and $s = \dfrac{y}{4}$ because x is the circumference of the circle and y is the perimeter of the square, thus $A = \dfrac{x^2}{4\pi} + \dfrac{y^2}{16}$. But $x + y = 12$, so $y = 12 - x$ and

$$A = \frac{x^2}{4\pi} + \frac{(12-x)^2}{16} = \frac{\pi + 4}{16\pi}x^2 - \frac{3}{2}x + 9 \text{ for } 0 \le x \le 12.$$

$\dfrac{dA}{dx} = \dfrac{\pi + 4}{8\pi}x - \dfrac{3}{2}$, $\dfrac{dA}{dx} = 0$ when $x = \dfrac{12\pi}{\pi + 4}$. If $x = 0, \dfrac{12\pi}{\pi + 4}, 12$ then $A = 9, \dfrac{36}{\pi + 4}, \dfrac{36}{\pi}$ so the sum of the enclosed areas is

(a) a maximum when $x = 12$ in. (when all of the wire is used for the circle)

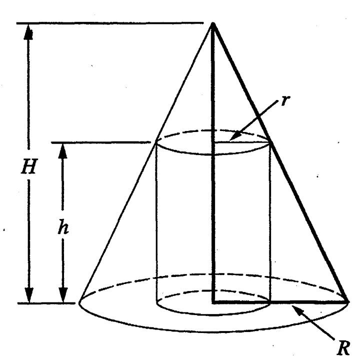
(b) a minimum when $x = 12\pi/(\pi + 4)$ in.



19. Let x be the length of each side of a square, then $V = x(3 - 2x)(8 - 2x) = 4x^3 - 22x^2 + 24x$ for $0 \le x \le 3/2$; $dV/dx = 12x^2 - 44x + 24 = 4(3x - 2)(x - 3)$, $dV/dx = 0$ when $x = 2/3$ for $0 < x < 3/2$. If $x = 0, 2/3, 3/2$ then $V = 0, 200/27, 0$ so the maximum volume is $200/27$ ft^3.

20. Let $x =$ length of each edge of base, $y =$ height. The cost is $C =$ (cost of top and bottom) $+$ (cost of sides) $= (2)(2x^2) + (3)(4xy) = 4x^2 + 12xy$, but $V = x^2y = 2250$ thus $y = 2250/x^2$ so $C = 4x^2 + 27000/x$ for $x > 0$, $dC/dx = 8x - 27000/x^2$, $dC/dx = 0$ when $x = \sqrt[3]{3375} = 15$, $d^2C/dx^2 > 0$ so C is least when $x = 15$, $y = 10$.

26. Let R and H be the radius and height of the cone, and r and h the radius and height of the cylinder (see figure), then the volume of the cylinder is $V = \pi r^2 h$. By similar triangles (see figure) $\dfrac{H - h}{H} = \dfrac{r}{R}$ thus $h = \dfrac{H}{R}(R - r)$ so $V = \pi\dfrac{H}{R}(R - r)r^2 = \pi\dfrac{H}{R}(Rr^2 - r^3)$ for $0 \le r \le R$. $\dfrac{dV}{dr} = \pi\dfrac{H}{R}(2Rr - 3r^2) = \pi\dfrac{H}{R}r(2R - 3r)$, $\dfrac{dV}{dr} = 0$ for $0 < r < R$ when $r = 2R/3$. If $r = 0, 2R/3, R$ then $V = 0, 4\pi R^2H/27, 0$ so the maximum volume is $\dfrac{4\pi R^2 H}{27} = \dfrac{4}{9}\dfrac{1}{3}\pi R^2 H = \dfrac{4}{9}\cdot$ (volume of cone).



29. The surface area is $S = \pi r^2 + 2\pi rh$ where $V = \pi r^2 h = 500$ so $h = 500/(\pi r^2)$ and $S = \pi r^2 + 1000/r$ for $r > 0$; $dS/dr = 2\pi r - 1000/r^2 = (2\pi r^3 - 1000)/r^2$, $dS/dr = 0$ when $r = \sqrt[3]{500/\pi}$, $d^2S/dr^2 > 0$ for $r > 0$ so S is minimum when $r = \sqrt[3]{500/\pi}$ cm and

$$h = \frac{500}{\pi r^2} = \frac{500}{\pi}\left(\frac{\pi}{500}\right)^{2/3} = \sqrt[3]{500/\pi} \text{ cm}$$