30. $u = 4 - x$, $du = -dx$;

$$-\int(4-u)\sqrt{u}\,du = -\frac{8}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C = \frac{2}{5}(4-x)^{5/2} - \frac{8}{3}(4-x)^{3/2} + C$$

31. $\int \sin^2 2\theta \sin 2\theta\, d\theta = \int (1 - \cos^2 2\theta)\sin 2\theta\, d\theta$; $u = \cos 2\theta$, $du = -2\sin 2\theta\, d\theta$,

$$-\frac{1}{2}\int(1-u^2)du = -\frac{1}{2}u + \frac{1}{6}u^3 + C = -\frac{1}{2}\cos 2\theta + \frac{1}{6}\cos^3 2\theta + C$$

32. $\sec^2 3\theta = \tan^2 3\theta + 1, u = 3\theta, du = 3d\theta$

$$\int \sec^4 3\theta\, d\theta = \frac{1}{3}\int(\tan^2 u + 1)\sec^2 u\, du = \frac{1}{9}\tan^3 u + \frac{1}{3}\tan u + C = \frac{1}{9}\tan^3 3\theta + \frac{1}{3}\tan 3\theta + C$$

33. $u = a + bx, du = b\,dx$,

$$\int(a+bx)^n\, dx = \frac{1}{b}\int u^n du = \frac{(a+bx)^{n+1}}{b(n+1)} + C$$

34. $u = a + bx$, $du = b\,dx$, $dx = \frac{1}{b}du$

$$\frac{1}{b}\int u^{1/n}du = \frac{n}{b(n+1)}u^{(n+1)/n} + C = \frac{n}{b(n+1)}(a+bx)^{(n+1)/n} + C$$

35. $u = \sin(a+bx)$, $du = b\cos(a+bx)dx$

$$\frac{1}{b}\int u^n du = \frac{1}{b(n+1)}u^{n+1} + C = \frac{1}{b(n+1)}\sin^{n+1}(a+bx) + C$$

37. **(a)** with $u = \sin x$, $du = \cos x\, dx$; $\int u\, du = \frac{1}{2}u^2 + C_1 = \frac{1}{2}\sin^2 x + C_1$;

with $u = \cos x$, $du = -\sin x\, dx$; $-\int u\, du = -\frac{1}{2}u^2 + C_2 = -\frac{1}{2}\cos^2 x + C_2$

(b) because they differ by a constant:

$$\left(\frac{1}{2}\sin^2 x + C_1\right) - \left(-\frac{1}{2}\cos^2 x + C_2\right) = \frac{1}{2}(\sin^2 x + \cos^2 x) + C_1 - C_2 = 1/2 + C_1 - C_2$$

38. **(a)** First method: $\int(25x^2 - 10x + 1)dx = \frac{25}{3}x^3 - 5x^2 + x + C_1$;

second method: $\frac{1}{5}\int u^2 du = \frac{1}{15}u^3 + C_2 = \frac{1}{15}(5x-1)^3 + C_2$

(b) $\frac{1}{15}(5x-1)^3 + C_2 = \frac{1}{15}(125x^3 - 75x^2 + 15x - 1) + C_2 = \frac{25}{3}x^3 - 5x^2 + x - \frac{1}{15} + C_2$;

the answers differ by a constant.

39. $y = \int\sqrt{5x+1}\,dx = \frac{2}{15}(5x+1)^{3/2} + C; -2 = y(3) = \frac{2}{15}64 + C$,

so $C = -2 - \frac{2}{15}64 = -\frac{158}{15}$, and $y = \frac{2}{15}(5x+1)^{3/2} - \frac{158}{15}$

40. $y = \int(2 + \sin 3x)\, dx = 2x - \frac{1}{3}\cos 3x + C$ and

$0 = y\left(\frac{\pi}{3}\right) = \frac{2\pi}{3} + \frac{1}{3} + C, C = -\frac{2\pi+1}{3}, y = 2x - \frac{1}{3}\cos 3x - \frac{2\pi+1}{3}$