

41.  $u = 4x$ , use equation (19) to get

$$\frac{1}{4} \int \tan^3 u \, du = \frac{1}{4} \left[ \frac{1}{2} \tan^2 u + \ln |\cos u| \right] + C = \frac{1}{8} \tan^2 4x + \frac{1}{4} \ln |\cos 4x| + C$$

42. Use equation (19) to get  $\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x - \tan x + x + C$

$$43. \int \sqrt{\tan x} (1 + \tan^2 x) \sec^2 x \, dx = \frac{2}{3} \tan^{3/2} x + \frac{2}{7} \tan^{7/2} x + C$$

$$44. \int \sec^{1/2} x (\sec x \tan x) \, dx = \frac{2}{3} \sec^{3/2} x + C$$

47.  $u = x/2$ ,

$$2 \int_0^{\pi/4} \tan^5 u \, du = \left[ \frac{1}{2} \tan^4 u - \tan^2 u - 2 \ln |\cos u| \right]_0^{\pi/4} = 1/2 - 1 - 2 \ln(1/\sqrt{2}) = -1/2 + \ln 2$$

$$48. u = \pi x, \frac{1}{\pi} \int_0^{\pi/4} \sec u \tan u \, du = \frac{1}{\pi} \sec u \Big|_0^{\pi/4} = (\sqrt{2} - 1)/\pi$$

$$49. \int (\csc^2 x - 1) \csc^2 x (\csc x \cot x) \, dx = \int (\csc^4 x - \csc^2 x) (\csc x \cot x) \, dx = -\frac{1}{5} \csc^5 x + \frac{1}{3} \csc^3 x + C$$

$$50. \int \frac{\cos^2 3t}{\sin^2 3t} \cdot \frac{1}{\cos 3t} \, dt = \int \csc 3t \cot 3t \, dt = -\frac{1}{3} \csc 3t + C$$

$$51. \int (\csc^2 x - 1) \cot x \, dx = \int \csc x (\csc x \cot x) \, dx - \int \frac{\cos x}{\sin x} \, dx = -\frac{1}{2} \csc^2 x - \ln |\sin x| + C$$

$$52. \int (\cot^2 x + 1) \csc^2 x \, dx = -\frac{1}{3} \cot^3 x - \cot x + C$$