4. $x = 3\sin\theta,\ dx = 3\cos\theta\,d\theta,$

$$\frac{1}{9}\int \frac{1}{\sin^2\theta}d\theta = \frac{1}{9}\int \csc^2\theta\,d\theta = -\frac{1}{9}\cot\theta + C = -\frac{\sqrt{9-x^2}}{9x} + C$$

5. $x = 2\tan\theta,\ dx = 2\sec^2\theta\,d\theta,$

$$\frac{1}{8}\int \frac{1}{\sec^2\theta}d\theta = \frac{1}{8}\int \cos^2\theta\,d\theta = \frac{1}{16}\int (1+\cos 2\theta)d\theta = \frac{1}{16}\theta + \frac{1}{32}\sin 2\theta + C$$

$$= \frac{1}{16}\theta + \frac{1}{16}\sin\theta\cos\theta + C = \frac{1}{16}\tan^{-1}\frac{x}{2} + \frac{x}{8(4+x^2)} + C$$

6. $x = \sqrt{5}\tan\theta,\ dx = \sqrt{5}\sec^2\theta\,d\theta,$

$$5\int \tan^2\theta\sec\theta\,d\theta = 5\int (\sec^3\theta - \sec\theta)d\theta = 5\left(\frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta|\right) + C_1$$

$$= \frac{1}{2}x\sqrt{5+x^2} - \frac{5}{2}\ln\frac{\sqrt{5+x^2}+x}{\sqrt{5}} + C_1 = \frac{1}{2}x\sqrt{5+x^2} - \frac{5}{2}\ln(\sqrt{5+x^2}+x) + C$$

7. $x = 3\sec\theta,\ dx = 3\sec\theta\tan\theta\,d\theta,$

$$3\int \tan^2\theta\,d\theta = 3\int (\sec^2\theta - 1)d\theta = 3\tan\theta - 3\theta + C = \sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C$$

8. $x = 4\sec\theta,\ dx = 4\sec\theta\tan\theta\,d\theta,$

$$\frac{1}{16}\int \frac{1}{\sec\theta}d\theta = \frac{1}{16}\int \cos\theta\,d\theta = \frac{1}{16}\sin\theta + C = \frac{\sqrt{x^2-16}}{16x} + C$$

9. $x = \sin\theta,\ dx = \cos\theta\,d\theta,$

$$3\int \sin^3\theta\,d\theta = 3\int \left[1-\cos^2\theta\right]\sin\theta\,d\theta$$

$$= 3\left(-\cos\theta + \cos^3\theta\right) + C = -3\sqrt{1-x^2} + (1-x^2)^{3/2} + C$$

10. $x = \sqrt{5}\sin\theta,\ dx = \sqrt{5}\cos\theta\,d\theta,$

$$25\sqrt{5}\int \sin^3\theta\cos^2\theta\,d\theta = 25\sqrt{5}\left(-\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta\right) + C = -\frac{5}{3}(5-x^2)^{3/2} + \frac{1}{5}(5-x^2)^{5/2} + C$$

11. $x = \frac{2}{3}\sec\theta,\ dx = \frac{2}{3}\sec\theta\tan\theta\,d\theta,\ \frac{3}{4}\int \frac{1}{\sec\theta}d\theta = \frac{3}{4}\int \cos\theta\,d\theta = \frac{3}{4}\sin\theta + C = \frac{1}{4x}\sqrt{9x^2-4} + C$

12. $t = \tan\theta,\ dt = \sec^2\theta\,d\theta,$

$$\int \frac{\sec^3\theta}{\tan\theta}d\theta = \int \frac{\tan^2\theta+1}{\tan\theta}\sec\theta\,d\theta = \int (\sec\theta\tan\theta + \csc\theta)d\theta$$

$$= \sec\theta + \ln|\csc\theta - \cot\theta| + C = \sqrt{1+t^2} + \ln\frac{\sqrt{1+t^2}-1}{|t|} + C$$

13. $x = \sin\theta,\ dx = \cos\theta\,d\theta,\ \int \frac{1}{\cos^2\theta}d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C = x/\sqrt{1-x^2} + C$